# Table of Contents

Editors’ Introduction  
HU FU and S. MATTHEW WEINBERG  
1

Letter from SIGecom Executive Committee  
NICOLE IMMORLICA, SCOTT DUKE KOMINERS, and KATRINA LIGETT  
2

Job Market Candidate Profiles 2020  
VASILIS GKATZELIS and JASON HARTLINE  
4

Informational Bounds on Equilibria  
YAKOV BABICHENKO  
25

Multi-dimensional Mechanism Design via Random Order Contention Resolution Schemes  
MAREK ADAMCZYK and MICHAL WLODARCZYK  
46

Tight Revenue Gaps among Simple and Optimal Mechanisms  
YAONAN JIN, PINYAN LU, QI QI, ZHIHAO GAVIN TANG, and TAO XIAO  
54

Sample Complexity of Single-parameter Revenue Maximization  
CHENGHAO GUO, ZHIYI HUANG, and XINZHI ZHANG  
62

Justifications of Welfare Guarantees under Normalized Utilities  
HARIS AZIZ  
71

Puzzle: The AI Circus  
VINCENT CONITZER  
76
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Editors’ Introduction

HU FU
University of British Columbia
and
S. MATTHEW WEINBERG
Princeton University

This issue of SIGecom Exchanges contains a letter from the SIGecom executive committee, the job market candidate profiles for 2020, a survey, three research letters, a short note, and a puzzle.

The 2020 SIGecom Job Candidates Profiles are again compiled by Jason Hartline and Vasilis Gkatzelis. This great service for the community has become an annual tradition for the Exchanges.

The survey contributed by Yakov Babichenko summarizes recent progress on query complexity and communication complexity of Nash equilibria, correlated equilibria, and coarse correlated equilibria.

Marek Adamczak and Michał Włodarczyk contributed a letter on their FOCS 18 paper on random order contention resolution schemes, with applications to Bayesian multi-parameter unit-demand mechanism design. Yaonan Jin, Pinyan Lu, Qi Qi, Zhihao Gavin Tang, and Tao Xiao contributed a letter on their STOC 19 paper on tight approximation ratio of anonymous pricing, which closes an open question in the pricing literature. Chenghao Guo, Zhiyi Huang, and Xinzhi Zhang contributed a letter on their STOC 19 paper on sample complexity of single-parameter revenue maximization, which in its turn closes open problems in sample complexity of auctions, with a novel technique using information theory.

The short note by Haris Aziz enumerates justifications for the assumption of normalized utility. Some justifications are better known in the community, while others considerably innovative.

Vincent Conitzer contributed a puzzle in honor of Tuomas Sandholm’s 50th birthday. Vincent also graciously offered to be the judge of submitted solutions, the best of which is to be published in the next issue of the Exchanges. Happy birthday, Tuomas!

This issue is the last with Hu as a co-editor. Starting next issue, Inbal Talgam-Cohen, Assistant Professor at Technion, will join Matt as a co-editor. Hu would like to thank all contributors in the past three years and the community in general for their support and help. Felix Fischer, our long-time Information Director, is also passing the baton to Yannai Gonczarowski. Thank you very much, Felix, for maintaining the website and publishing the Exchanges over the many years!

We hope you, the reader, will enjoy this issue.

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Letter from SIGecom Executive Committee

NICOLE IMMORLICA
Microsoft Research
and
SCOTT DUKE KOMINERS
Harvard University
and
KATRINA LIGETT
Hebrew University of Jerusalem

Dear SIGecom community,

We, the new SIGecom executives, are honored to be serving as your representatives. First, we’d like to heap praise and thanks on the outgoing executives – Kevin Leyton-Brown, Michal Feldman, and Jenn Wortman Vaughan – for their boundless energy, engagement, and wisdom in their four years of leadership. We have a lot to live up to.

We have identified three broad goals for the coming years: disciplinary and demographic diversity, a cohesive community, and most importantly, sustained and significant impact on academia and society. Our initiatives are guided by these goals, and we are in the process of developing metrics to track our progress.

We are proud that EC’19 had a record ≈600 attendees from over 30 different countries, with 22% identifying as female — not even counting remote attendees of the live-stream sites in Ethiopia, Kenya, Nigeria and Uganda (a special initiative run by Rediet Abebe and Eric Sodomka). And we’re looking forward to working with all of you to take EC and the SIG to the next level.

We have an amazing slate of people onboard to help lead SIG initiatives. Yannai Gonczarowski is serving as SIGecom Information Director, taking on the role from Felix Fischer. Yannai is hard at work expanding the SIG’s digital presence. Be on the lookout for an improved SIGecom website and an entry into social media. Michal Feldman is leading SafeEC, a subsidiary of SafeToC, helping to provide safe pathways to elevate ethics concerns. You can read more about this initiative — and find a list of advocates — at SafeToc.org. Please reach out to Michal or any SafeToC advocate if you would like to help in this effort, or if anything is troubling you. Vasilis Gkatzelis is Special Initiatives Chair, his first initiative being the job markets profiles published regularly in Exchanges (including this issue). If you have ideas for new initiatives that would further the goals of the SIG, please do contact him. Inbal Talgam-Cohen and Matt Weinberg (and Hu Fu before them) are the Exchanges editors, keeping us all informed of the community’s ideas and news.

Authors’ addresses: nicimm@gmail.com; kominers@fas.harvard.edu; katrina@cs.huji.ac.il
Last, but not least, Michael Ostrovsky and Ariel Procaccia are the Program Chairs, and Péter Biró the General Chair, of the upcoming EC’20, which will be co-located with GAMES in Budapest next summer from July 13-17. We hope to see you all there!

Your SIGecom executives,

Nicole Immorlica (chair), Scott Duke Kominers, and Katrina Ligett
SIGecom Job Market Candidate Profiles 2020

Edited by VASILIS GKATZELIS and JASON HARTLINE

This is the fifth annual collection of profiles of the junior faculty job market candidates of the SIGecom community. The seventeen candidates for 2020 are listed alphabetically and indexed by research areas that define the interests of the community. Along with information regarding the candidate’s bio, research summary, and representative papers, each profile also contains links to the candidate’s homepage, CV, and Google scholar publication profile.

Employers with a relevant job opening can reach all the listed candidates directly by sending the relevant information to the moderated mailing list ecom-candidates2020@acm.org.

—Vasillis Gkatzelis and Jason Hartline

Fig. 1. Generated using the research summaries of the candidates.

Contents

Navid Azizan
optimization, market design, pricing, machine learning, networks 6

Yuan Deng
mechanism design, dynamic auctions, online learning 7

Ludwig Dierks
market design, operations research, queuing theory, cloud computing 8

Rupert Freeman
information elicitation, fair division, social choice 9

Ophir Friedler
mechanism design, simple auctions, combinatorial auctions, market design 10

Nikhil Garg
market design, social choice, learning & data science, online platforms 11
<table>
<thead>
<tr>
<th>Name</th>
<th>Affiliations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aleck Johnsen</td>
<td>mechanism design, inference, online markets, prior independence</td>
</tr>
<tr>
<td>Bo Li</td>
<td>fair division, mechanism design, online algorithms, blockchain</td>
</tr>
<tr>
<td>Amin Rahimian</td>
<td>social networks, influence maximization, social learning, social contagion</td>
</tr>
<tr>
<td>Ariel Schwartzman</td>
<td>mechanism design, menu complexity, tournament design</td>
</tr>
<tr>
<td>Ali Shameli</td>
<td>mechanism design, matching markets, revenue management</td>
</tr>
<tr>
<td>Biaoshuai Tao</td>
<td>influence maximization, social networks, fair division, resource allocation</td>
</tr>
<tr>
<td>Yixin Tao</td>
<td>market efficiency and dynamics, asynchronous optimization, fair division</td>
</tr>
<tr>
<td>Alexandros Voudouris</td>
<td>resource allocation, price of anarchy, rank aggregation, peer grading</td>
</tr>
<tr>
<td>David Wajc</td>
<td>matching, online algorithms, dynamic algorithms</td>
</tr>
<tr>
<td>Fang-Yi Yu</td>
<td>social networks, dynamical systems, information elicitation</td>
</tr>
<tr>
<td>Manolis Zampetakis</td>
<td>statistics, machine learning, complexity theory, mechanism design</td>
</tr>
</tbody>
</table>
NAVID AZIZAN (Homepage, CV, Scholar)

Thesis: Optimization Algorithms for Non-Convex and Networked Systems

Advisor: Adam Wierman and Babak Hassibi, California Institute of Technology

Brief Biography: Navid Azizan is a PhD candidate in Computing and Mathematical Sciences (CMS) at the California Institute of Technology (Caltech), where he is co-advised by Adam Wierman and Babak Hassibi. During the summer of 2019, he was a research scientist intern at Google DeepMind. He received the B.Sc. degree form Sharif University of Technology and the M.Sc. degree from the University of Southern California, in 2013 and 2015, respectively. His research interests broadly lie in optimization, networks, markets, and machine learning. He is the recipient of several awards, including the 2016 ACM GreenMetrics Best Student Paper Award, the Amazon PhD Fellowship, and the PIMCO PhD Fellowship.

Research Summary: While convexity is a foundational assumption in much of the literature in both computing and economics, most systems are non-convex in practice, and not accounting for these non-convexities can lead to significant inefficiencies. The other challenging aspect of today’s large-scale systems is that they are networked. My research is aimed at developing new tools for solving non-convex and networked problems and applying these tools to real-world systems in various domains such as market design and machine learning.

One of the most important examples of non-convex problems in economics is that of pricing in non-convex markets (which is, for instance, of crucial importance to energy markets). When the costs are non-convex, there may be no linear prices that support a competitive equilibrium, and there is a large literature on how to determine prices in such markets. However, all the existing pricing schemes lack important economic properties. In [1], we provide a new approach to pricing in non-convex markets, which is applicable to general non-convex costs, and guarantees all the economic properties sought in the literature. Further, we provide a polynomial-time algorithm to compute the prices that works for a very general family of non-convex costs. Lastly, we show how to extend the result to networked markets.

Another important, and perhaps the most prevalent, non-convex problem of the modern era in computing is that of training deep neural networks. While these problems are non-convex in general, we prove in [3] that the huge number of parameters in typical deep models makes the training algorithm converge to a global minimum, and that in such settings, stochastic gradient descent acts as a particular form of regularizer. We further propose using an alternative algorithm, i.e., stochastic mirror descent, for training deep nets, which we prove induces a different form of regularization, and leads to improving the state of the art by a wide margin.

Representative Papers:


[3] Stochastic Gradient/Mirror Descent: Minimax Optimality and Implicit Regularization (ICLR’19, and NeurIPS’18 Workshop) with B. Hassibi
YUAN DENG (Homepage, CV, Scholar)

**Thesis:** Dynamic Mechanism Design in Complex Environments

**Advisor:** Vincent Conitzer, Duke University

**Brief Biography:** Yuan is a fifth-year PhD candidate at Duke University in Computer Science, where he is advised by Vincent Conitzer. Before that, he was an undergraduate student at Tsinghua University. He spent two summers interning in the market algorithms group at Google Research New York with Sébastien Lahaie and Vahab Mirrokni in 2018, and with Balasubramanian Sivan in 2019. He is a recipient of the 2018 Google PhD Fellowship in Algorithms, Optimizations and Markets, and James B. Duke Fellowship. His research is broadly situated at the interface between economics and computer science, and mainly seeks to understand how to design mechanisms in dynamic environments.

**Research Summary:** As a fundamental problem in mechanism design, pricing in repeated auctions has been extensively studied in recent years, partly motivated by the popularity of selling online ads via auctions. Dynamic auctions open up the possibility of linking auctions across time to achieve higher revenue and/or welfare than static auctions, but the complexity and reliance on exact distributional information limit the deployment of dynamic auctions in practice.

My research addresses two aspects of dynamic mechanism design. The first concerns how to design dynamic auctions when only the estimated distributional information is available to the seller. We introduce a framework for the design of mechanisms that are robust to both estimation errors in the distributional information and the buyers’ strategic behavior. We combine our framework and techniques from online learning to design auctions that estimate the distributional information on the fly and achieve low regret against the revenue-optimal dynamic auction [1].

The second thread explores how to design dynamic mechanisms with constraints, in particular, when the buyers have budget constraints. We characterize the simple structures of optimal dynamic mechanisms. Applying the characterization, we design a non-clairvoyant dynamic mechanism with budget constraints that achieves a constant approximation of the revenue-optimal dynamic auction [2].

Another line of my research aims to investigate how to strategize against no-regret agents. This line of research is partly motivated by online advertising markets, in which the buyers may use no-regret algorithms in bidding while the seller can exploit the buyers’ no-regret strategies to increase revenue. In general two-player normal-form games with one strategic optimizer and one no-regret learner, we present a tight characterization of what various properties of the learner’s no-regret algorithms imply for the optimizer’s best behavior [3]. We further extend these techniques in designing revenue-optimal auctions against no-regret buyers.

**Representative Papers:**

[1] A Robust Non-Clairvoyant Dynamic Mechanism for Contextual Auctions (NeurIPS'19) with S. Lahaie and V. Mirrokni

[2] Non-Clairvoyant Dynamic Mechanism Design with Budget Constraints and Beyond (SSRN) with V. Mirrokni and S. Zuo

[3] Strategizing against No-regret Learners (NeurIPS’19) with J. Schneider and B. Sivan
LUDWIG DIERKS (Homepage, CV, Scholar)

Thesis: Market Design for Cloud Computing

Advisor: Sven Seuken, University of Zurich

Brief Biography: Ludwig is a Ph.D. student advised by Prof. Sven Seuken in the Computation and Economics Research Group at the Department of Informatics of the University of Zurich. In Summer 2018, Ludwig was an intern at Microsoft in the Office of the Chief Economist. In 2015, he received a M.Sc. in Mathematics in Operations Research from TU Munich. He wrote his Master's thesis on "Cooperative Games with Payoff Restrictions" under the supervision of Prof. Felix Brandt. During his M.Sc. he visited the NCKU in Tainan, Taiwan (R.O.C.), for one year as an exchange student under the supervision of Prof. Ruey-Lin Sheu.

Research Summary: In my research, I employ a variety of market design and operations research techniques to identify and analyze inefficiencies in existing markets and to develop mechanisms that mitigate or avoid them. During my PhD, my work was mostly focused on the domain of cloud computing.

Large amounts of capacity in cloud computing centers often stand idle because they are reserved for tasks that do not actually use them at all times (e.g., maintenance, or users with long-term contracts). A natural idea to increase a cloud provider's profit is to sell this idle capacity on a secondary market, in the form of preemptible capacity that can be taken back at any time. In a first paper [1], we focus on preemptible spot markets, i.e., markets where users can directly bid for capacity. We model the provider's profit optimization problem by combining queuing theory and game theory to analyze the equilibria of the resulting queuing system. The main result is an easy-to-check condition under which a provider can simultaneously achieve a profit increase and create a Pareto improvement for the users by offering a spot market (using idle resources) alongside a fixed-price market. In an as of yet unpublished second paper, we extend the analysis to secondary fixed-price markets and compare both secondary market types.

In a second research strand, I focused on the cluster admission control problem: Many modern cloud workload are characterized by scaling resource demands. A provider therefore has to continuously decide whether she can add additional workloads to a given compute cluster or if doing so would impact existing workloads’ ability to scale. In [2], we formalize the problem as a constrained partially observable Markov decision process (POMDP). As no way to feasibly solve this POMDP is known, we systematically relax it to design quick heuristic admission policies. These policies estimate moments of each workload’s distribution of future resource usage. Through simulations we evaluate the performance of our policies compared to current industry standards. We further evaluate by how much utilization can be improved with learned or elicited prior information and show how to incentivize users to provide this information.

Representative Papers:


[2] On the cluster admission problem for cloud computing (NetEcon’19) with I. Kash and S. Seuken
RUPERT FREEMAN (Homepage, CV, Scholar)

Thesis: Eliciting and Aggregating Information for Better Decision Making

Advisor: Vincent Conitzer, Duke University

Brief Biography: Rupert Freeman is a postdoc at Microsoft Research New York City. Previously, he received his Ph.D. from Duke University under the supervision of Vincent Conitzer. His research focuses on topics such as resource allocation, social choice, and information elicitation. He is the recipient of a Facebook Ph.D. Fellowship and a Duke Computer Science outstanding dissertation award.

Research Summary: My research focuses on problems where the goal is to elicit private information (like probabilistic judgments or subjective preferences) and aggregate it into a single output (like a forecast, action, or resource allocation). I am most excited by problems and solutions that draw on a variety of techniques from computer science and economics such as auctions and market design, game theory, algorithm design, artificial intelligence, and machine learning.

In a recent line of work, we have been exploring connections between wagering mechanisms — a class of elicitation mechanisms grounded in the classic economic theory of proper scoring rules — and other seemingly unrelated problems. For instance, a reinterpretation of outcome-contingent payments as an allocation of Arrow-Debreu securities yields a surprising equivalence between wagering and a natural resource allocation problem where agents have additive preferences over a set of divisible items [1]. The equivalence immediately yields new mechanisms and results for both settings. In other work [2], we use repeated application of a wagering mechanism to design the first incentive-compatible mechanism for selecting the winner of a forecasting competition, such as the Netflix Prize or a Kaggle competition. In an ongoing project [3], we have found that similar ideas can be used to design algorithms for learning from expert advice that are incentive compatible when experts care about the weight assigned to them by the learning algorithm, without suffering any loss in terms of regret guarantees.

In an EC 2019 paper [4], we consider a social choice problem where we are required to aggregate a set of individual budget proposals into a single proposal. We present a solution to this problem by drawing a connection between prices in a simple market and generalized median mechanisms. Although markets do not generally yield incentive-compatible mechanisms for resource allocation, they provide us with a powerful tool for incentive-compatible preference aggregation through their prices.

Representative Papers:


OPHIR FRIEDLER (Homepage, CV, Scholar)

**Thesis:** Simple Mechanisms for Complex Environments

**Advisor:** Michal Feldman, Tel Aviv University

**Brief Biography:** Ophir is a fifth-year PhD student at the Computer Science Department in Tel Aviv University, under the supervision of Michal Feldman, where he also obtained his MSc. Prior to his MSc, Ophir obtained his BSc at the Technion - Israel Institute of Technology.

**Research Summary:** My main research interests are in Algorithmic Game Theory and Mechanism Design. During my PhD, I study various types of simple mechanisms, in complex settings where optimal mechanisms are poorly understood, intractable, or cannot be realistically implemented.

I put an emphasis on less standard settings such as agents with valuations that may admit complementarities across items (and not only substitutes). In [1], we study previously studied (EC 2015) simple mechanisms, which were analyzed for agents with complement-free valuations. In [1] we prove such mechanisms have good social welfare guarantees for valuations with limited degree of complementarity. Moreover, we provide different but similar mechanisms with improved guarantees for general valuations. For the revenue objective, in [2] we show that the better mechanism between selling items separately, or selling the grand bundle of all items, has good revenue guarantees in settings with limited degree of complementarity.

In an EC 2017 paper, we study competition complexity – the number of extra bidders needed for a prior free mechanism, to extract at least as much revenue as the optimal, prior-based mechanism. We show that for additive bidders with feasibility constraints, the prior free VCG mechanism with a linear number of extra bidders, extracts at least as much revenue as the optimal, poorly understood, computationally intractable mechanism. In [3] we slightly relax the objective from the optimal revenue, to 99% of the optimal revenue. We provide a host of results showing that for additive bidders, 99% of the revenue can be achieved, by using substantially less (to no extra) competition.

In an ongoing work [4], we study Walrasian equilibrium, subject to the endowment effect, where consumers tend to inflate the value of items they own. Following up on an EC 2018 work, we provide a more flexible formulation of the endowment effect and prove stability results with approximate efficiency for generalized settings, as well as settings where the market designer can pre-pack items to indivisible bundles.

**Representative Papers:**

[1] Simple Mechanisms for Agents with Complements (EC 2016)  
with M. Feldman, J. Morgenstern, and G. Reiner

with A. Eden, M. Feldman, I. Talgam-Cohen, and S.M. Weinberg

with M. Feldman and A. Rubinstein

with T. Ezra and M. Feldman
NIKHIL GARG (Homepage, CV, Scholar)

**Thesis:** Learning and Pricing in Human-centric Platforms

**Advisors:** Ashish Goel & Ramesh Johari, Stanford University

**Brief Biography:** I am a PhD candidate at Stanford University, where I am part of the Stanford Crowdsourced Democracy Team and the Society and Algorithms Lab. I use tools from across computer science, probability, and economics to study online platforms. I received my MS in Electrical Engineering from Stanford, and a BS in Computer Engineering and a BA in Plan II (Liberal Arts) from the University of Texas at Austin. I have interned at Uber, NASA, Microsoft, the Texas Senate, and IEEE’s policy arm, and am a NSF Graduate Research Fellow and McCoy Family Center for Ethics in Society Graduate Fellow.

**Research Summary:** Human-centric platforms increasingly mediate interactions between people, at their best enabling fair and efficient agreements within large, diverse groups. Principled design of such platforms connects behavioral considerations to more classical Econ-CS challenges such as defining and achieving appropriate objectives; understanding statistical and learning-theoretic limitations; and leveraging experiments and extant data. So far, I have worked on designing online marketplaces [1,2] and civic engagement platforms [3], focusing on mechanisms that efficiently learn heterogeneous participant preferences and price goods accordingly. In all my work I aim to bridge the gap between coarse theoretical insights and the fine-grained questions practitioners must answer, driven both by theory and data. My work has informed deployments at Uber, a large online labor platform, and in participatory budgeting elections in several U.S. cities.

In my work with Hamid Nazerzadeh [1], we consider how to design surge (dynamic) pricing in ride-hailing platforms, which is used to balance the supply of available drivers with the demand for rides. We show that due to the temporal dynamics of surge, trips of different time lengths vary in the opportunity cost they impose on drivers, and so some drivers may strategically reject trip requests to maximize their earnings, to the detriment of other drivers. We develop an incentive compatible pricing scheme to resolve this issue, with a simple enough closed-form expression to enable transparency and communication of surge prices through a heat-map. Finally, with calibrated numerics and empirical analysis, we compare additive vs multiplicative surge in practice.

**Representative Papers:**

[1] Driver Surge Pricing (In Submission)  
with H. Nazerzadeh

(In Submission) with R. Johari

[3] Iterative Local Voting for Collective Decision-making in Continuous Spaces  
(JAIR 2019 & WWW 2017)  
with V. Kamble, A. Goel, D. Marn, and K. Munagala

ALECK JOHNSEN (Homepage, CV)

**Thesis:** Bid-Inversion Mechanisms and Robustness Lower Bounds

**Advisor:** Jason Hartline, Northwestern University

**Brief Biography:** Working in industry before graduate school, Aleck has accumulated 10 years of experience in trading, covering most roles: floor clerk and trader, algorithmic trader, algorithm designer, data scientist, and programmer; involving equities, fixed income, and options. He has a CFA charter and has done investor-side venture analysis of startups. During early graduate summers, Aleck wrote a C# e-file module (for transmitting to the IRS) for Forte International Tax (Evanston, IL). For 12 weeks (2016), Aleck visited booking.com (Amsterdam) with advisor Jason Hartline and colleague Dr. Denis Nekipelov (Virginia), where they helped booking’s ranking team apply principles of auction theory. At Northwestern, Aleck taught EECS 214 Data Science and Data Management. His undergraduate degrees were in civil engineering; and mathematics; at University of Illinois (Urbana-Champaign).

**Research Summary:** Since visiting booking.com, Aleck’s research has focused on auction inversion techniques, covering both “online” implementation theory and “offline” inference. The major questions are as follows. Can we design a predictive interface (e.g., a dashboard) such that we get both agents wanting to best respond to predictions, and we can infer values to run a desired social choice rule?[1] Can we design a mapping from non-truthful agent bids back to private values such that when we implement a desired choice rule, the side-effects (e.g., payments) are consistent with the original agent reports?[forthcoming] And, can we infer agents’ private values in auctions from the allocation rule and the prices they pay?[2]

Aleck has further (unpublished) work on prior independent design and benchmark design for worst case analysis. For prior independence; if a correlated distribution (over n variables) can be “decomposed” in two distinct ways—i.e., into distinct distributions over i.i.d. product distributions—then a lower bound on the performance follows as a consequence. A small set of meaningful examples of such “unidentified” correlated distributions (with decompositions) have been discovered. These results can extend to online algorithm settings, not just auctions. Within revenue auctions, if the optimal prior independent mechanism can be discovered—even for n = 2—and happens to respect a specific structure, as a corollary it will also solve an open question of Hartline-Roughgarden regarding “optimal” prior free benchmarks.

Aleck’s other interests include the interaction of game theory and big data, e.g. the bias-variance game studied in [3]; and results towards optimal expert learning with fixed time horizon.

**Representative Papers:**

   with J. Hartline, D. Nekipelov, O. Zoeter

[2] Inference from Auction Prices (SODA 2020; arxiv)
   with J. Hartline, D. Nekipelov, Z. Wang

[3] Bias-Variance Games (arxiv)
   with Y. Feng, R. Gradwohl, J. Hartline, D. Nekipelov
**BO LI** (Homepage, CV)

**Thesis:** Mechanism Design with Unstructured Information

**Advisor:** Jing Chen, Stony Brook University

**Brief Biography:** Bo Li is a postdoctoral researcher in the Department of Computer Science at University of Oxford, hosted by Edith Elkind. He received his PhD in 2019 from the Department of Computer Science at Stony Brook University, under the supervision of Jing Chen. He is broadly interested in algorithms, AI and computational economics, including problems related to fair division, mechanism design, online algorithms, and their applications to Blockchain. During his PhD, he spent a summer as a visiting student at ITCS (SUFE) with Pinyan Lu, a winter as a research assistant at ICT (CAS) with Xiaoming Sun, a summer as a research assistant at CityU with Minming Li, and a summer as a research intern at Algorand with Jing Chen. He is a recipient of the Catacosinos Fellowship, the Special Department Chair Fellowship, and a Sigma Xi Award. He completed his B.S. in Applied Maths and M.S. in Operations Research at Ocean University of China.

**Research Summary:** Fairly dividing a number of items among agents is an important topic in multi-agent system and AI. It has many applications in practice such as sharing rents and distributing goods/tasks. Part of my research is focused on a fundamental problem: how to characterize the fairness and efficiently compute such fair divisions for various situations. For mechanism design, I study how to design resilient mechanisms that work properly even in less foreseeable environments, such as when the mechanism designer’s information is imprecise or less structured, or when the computation/communication ability of the agents are limited.

With the emergence of many online platforms and dynamic data, I also study fair division and mechanism design problems in online settings, which capture many real-world scenarios, like resource sharing in data centers and labor crowdsourcing. We study to what extent the resource can be fairly divided among a number of online agents with different preferences, and how to disperse a number of online facilities to serve the demands uniformly distributed in a metric space. Furthermore, we show how to simultaneously achieve truthfulness and fairness in a 2-sided market with one side being processors and the other being online customers.

Recently, I have become interested in many game-theoretic questions raised in blockchain and bitcoin. For example, as bitcoin has been proved to be vulnerable for several attacks, we show how to refine the protocol so that participants do not launch these attacks in a Nash equilibrium. Another problem faced by blockchain is the lack of motivation for the participants to relay their information in the communication network, and in a working paper, we design a free market where strong incentives are provided to the users to fully propagate their information.

**Representative Papers:**


[3] Bayesian Auctions with Efficient Queries (working paper) with J. Chen, Y. Li, and P. Lu
AMIN RAHIMIAN (Homepage, CV, Scholar)

Thesis: Faster and Further Spreads in Social Networks

Advisor: Elchanan Mossel and Dean Eckles, MIT

Brief Biography: I am a postdoctoral associate at MIT Institute for Data, Systems, and Society (IDSS), co-advised by Elchanan Mossel (MIT Math) and Dean Eckles (MIT Sloan). I did my PhD in Electrical and Systems Engineering at the University of Pennsylvania, advised by Ali Jadbabaie. Broadly speaking my works are at the intersection of networks, data and decision sciences. I borrow tools from applied probability, statistics, algorithms, as well as decision and game theory. I am mostly interested in applications involving social and economic networks.

Research Summary:

In [1], we consider the choice of $k$ seeds in a social network to maximize the expected spread size. Most of the previous work on this problem (known as influence maximization) focuses on efficient algorithms to approximate the optimal seed sets with provable guarantees, assuming the knowledge of the entire network graph. However, in practice, obtaining full knowledge of the network structure is very costly. To address this gap, we propose algorithms that make a bounded number of queries to the graph structure and provide almost tight approximation guarantees.

In [2], we study how interventions that change the network structure can increase the speed of spread. For simple models in which contagion spreads through each edge independently at random, interventions that randomly rewire the edges would increase the speed of spread. However, for other contagion models that require multiple exposures before adoption (i.e. threshold-based contagions), recent work has argued for the opposite conclusion: highly clustered, rather than random, networks facilitate spread. In [2], we characterize the conditions under which we can reverse the latter result by allowing a small ($o(1)$) probability of sub-threshold adoptions.

In [3], we study the computations that Bayesian agents undertake when exchanging opinions over a network. The agents act repeatedly on their private information and take myopic actions that maximize their expected utility according to a fully rational posterior belief. We show that distinguishing between posteriors that are concentrated on different states of the world is NP-hard. Therefore, even approximating the Bayesian posterior beliefs is hard. We also describe a natural search algorithm to compute agents’ actions and beliefs, which we call elimination of impossible signals. We show that if the network is transitive, this algorithm can be modified to run in polynomial time.

Representative Papers:


ARIEL SCHVARTZMAN COHENCA (Homepage, CV)

Thesis: Circumventing Impossibility Results in Mechanism Design

Advisor: S. Matthew Weinberg, Princeton University

Brief Biography: Ariel Schwartzman Cohenca is a PhD candidate at Princeton University advised by S. Matthew Weinberg. Ariel’s work focuses in understanding the trade-off between optimality and simplicity in the design of multi-dimensional auctions. He was awarded the Department of Computer Science’s Graduate Student Teaching Award in 2017, and the School of Engineering and Applied Science’s Award for Excellence in 2018. During the summer of 2018, Ariel was a research intern at Google-Mountain View under the supervision of Gagan Aggarwal. He obtained his B.S. in Mathematics with Computer Science from MIT in 2015.

Research Summary:
Optimal mechanism design beyond single-item settings remains a central question at the intersection of economics and computer science. The problem is intricate for a number of reasons: the mechanisms may be bizarre, computationally hard to find or simply too complex to present to a bidder. The community’s focus, thus, has shifted from to asking, for instance, how complex must a mechanism be in order to extract 99% of the optimal revenue? My work joins that of others in quantifying this trade-off explicitly. Our results suggest that significantly simpler mechanisms can compete with optimal ones if the seller is willing to lose 1% of the optimal revenue (Kothari et al., FOCS 2019, Saxena et al., SODA 2018).

In settings where buyers have correlated valuations simple (or even finite) mechanisms have no hope of competing with optimal ones, even approximately. In light of this, we begin the study of beyond-worst case approximations for correlated bidders via the smoothed-analysis framework. Our results suggest ways to overcome long-standing impossibility results and shed light on the properties that make correlated distributions inapproximable (Psomas et al., EC 2019).

Finally, I am also interested in mechanism design for tournaments: how should a tournament designer pick a reasonable winner from a set of teams? We give an elegant answer to this question, showing that simple tournament formats are optimal among all fair ones that dissuade collusion (Schneider et al., ITCS 2017).

Representative Papers:

ALI SHAMELI (Homepage, CV)

**Thesis:** Algorithm Design in Online and Matching Markets

**Advisor:** Amin Saberi, Stanford University

**Brief Biography:** Ali is a fifth-year Ph.D. student at Stanford university at the MS&E department where he is advised by Amin Saberi. During his Ph.D. Ali did 3 internships at Adobe Research, Google Research, and Microsoft Research. He is the recipient of Stanford Graduate Fellowship, and the recipient of the best paper award in WINE 2017. He is also a visitor at the Simons Institute during the Fall of 2019. Ali is particularly interested in the study of online and matching markets.

**Research Summary:** The recent explosion in online and two sided marketplaces such as Uber, Airbnb, and Amazon Mturk, emphasizes the importance of better understanding of such platforms. These platforms have various objectives which are related to maximizing revenue or welfare of the market. For example, some online social networks aim to maximize information diffusion [3] as a means to increase welfare, or some matching markets face complicated challenges in providing high quality matches for users [1]. I am interested in identifying, modeling, analyzing, and providing a better understanding of the various challenges that these platforms face toward achieving their goals. Below I provide some examples.

One of the most well studied problems in mechanism design is the school choice problem. Although variations of this model are very interesting, even making small changes to this problem renders it too hard to solve. This motivated me to study the school choice problem subject to lower and upper bound quotas. This is a very important question since it allows us to enforce diversity in schools. In our OR paper [1], we efficiently solve this problem while approximately satisfying all the distributional constraints. Our algorithm is very general and has applications in various settings such as refugee assignment, or resident matching.

Motivated by the logistical challenges faced by different firms in their production lines, we defined and tackled the production constrained bayesian selection problem [2]. We modeled this problem as an Online Bayesian Selection problem subject to laminar matroid constraints. Our main result was a PTAS which we obtained by characterizing, relaxing, and rounding the LP for the optimal online policy. Our techniques are of independent interest and have applications in other settings such as selling flight itineraries or products with inventory replenishment.

More recently, I have been studying crowdsourcing platforms at Microsoft Research. What peaked my interest about this topic was that these platforms are intrinsically very inefficient from a mechanism design perspective since both sides of the market have an incentive to circumvent the platform to avoid paying any additional fees. This raises problems of great theoretical and practical interest.

**Representative Papers:**


[3] Information Aggregation in Overlapping Generations (WINE’17 (Best Paper)) with M. Akbarpour, and A. Saberi
BIAOSHUAI TAO (Homepage, CV, Scholar)

**Thesis:** Complexity, Algorithms, and Heuristics of Influence Maximization

**Advisor:** Grant Schoenebeck, University of Michigan

**Brief Biography:** I am currently working toward the Ph.D. degree with the Computer Science and Engineering Division, University of Michigan, Ann Arbor. I received the B.S. degree in mathematical science with a minor in computing from Nanyang Technological University in 2012.

**Research Summary:** My research focuses on the computational complexity and algorithm analysis aspects of economics problems. While I mainly work on social network analyses and resource allocation problems, I also work on other problems.

My primary research area is the *influence maximization problem*. In summary, my contribution to the *influence maximization problem* literature is two-fold. Firstly, I proved several novel and fundamental results about the algorithmic complexity of submodular influence maximization, some of which have been opened for more than 15 years. For example, I showed that influence maximization for well-studied cascade models, including the independent cascade model and the linear threshold model, is APX-hard, even if the networks are undirected [1]. Secondly, I proposed and studied a few new sociologically founded nonsubmodular cascade models and showed how they give fundamentally different recommendations to the seed-pickers in contrast to the submodular case [2, 3].

I have also been working on resource allocation problems, in particular, the *cake cutting problem*. *Fairness* is the most fundamental solution concept and is proved to be achievable in general, and my research focuses on the possibility and the algorithmic complexity of other solution concepts, such as *efficiency* and *strategy-proofness*, on top of that *fairness* is ensured. I showed that the problem of maximizing the social welfare (efficiency) while guaranteeing fairness is NP-hard to approximate to within a factor of $\Omega(\sqrt{n})$, and this problem admits a PTAS if agents’ utilities are linear [5]. This result has already appeared in textbooks. I also studied the cake cutting problem in a game theory setting where agents can misreport their utility functions on the cake. I proved that i) strategy-proofness and fairness cannot be simultaneously obtained when the number of agents is finite and ii) they can be obtained if the size of the market tends to infinity [4].

**Representative Papers:**

[1] Influence Maximization on Undirected Graphs: Towards Closing the $(1 - 1/e)$ Gap (EC’19) with G. Schoenebeck


[3] Beyond Worst-Case (In)approximability of Nonsubmodular Influence Maximization (ACM:TOCT’19 and WINE’17) with G. Schoenebeck


YIXIN TAO (Homepage, CV)

**Thesis:** Market Efficiency, Dynamics, and Optimization

**Advisor:** Richard Cole, New York University

**Brief Biography:** My BS degree in Computer Science was from the ACM Honor Class at Shanghai Jiao Tong University. Now, I am pursuing a Ph.D. in the Computer Science Department, Courant Institute of Mathematical Sciences, at NYU.

**Research Summary:** My research focuses on algorithmic game theory (AGT) and optimization. In AGT, my work mainly concerns market efficiency and market dynamics. I am also interested in fair division. In optimization, my work addresses asynchronous implementations of coordinate descent.

Market Dynamics: A major goal in AGT is to justify equilibrium concepts from a complexity perspective. One appealing approach is to identify natural distributed algorithms that converge quickly to an equilibrium. In the Fisher Market setting, we established new convergence results for two generalizations of Proportional Response when buyers have CES utility functions. The starting points are new convex and convex-concave formulations of such markets. The two generalizations correspond to suitable mirror descent algorithms applied to these formulations. Our results follow from new notions of strong Bregman convexity and of strong Bregman convex-concave functions, and associated linear rates of convergence.

Fair Division: Pareto Efficiency and envy-freeness are the foremost notions of, respectively, efficiency and fairness for the allocation problem. The question of whether there exists an allocation that is both Pareto Efficient and envy-free has been studied for a long time. Unfortunately, for general utility functions, in both the divisible and indivisible cases, solutions that are simultaneously Pareto Efficient and envy-free cannot be ensured. Our work focus on the indivisible case. We show that for any cardinal utility functions (including complementary utilities for example) and for any number of items and players, there always exists an ex-ante mixed allocation which is envy-free and Pareto Efficient.

Asynchronous Optimization: Coordinate descent is a core tool in machine learning and elsewhere. Large problem instances are common. To help solve them, two orthogonal approaches are known: acceleration and parallelism. Asynchronous parallel algorithms are appealing as they reduce the need for waiting, albeit at the cost of having to cope with out-of-date data. In our work, we give a comprehensive analysis of Asynchronous Stochastic Accelerated Coordinate Descent. We show: A linear speedup for strongly convex functions so long as the parallelism is not too large; a substantial, albeit sublinear, speedup for strongly convex functions for larger parallelism; a substantial, albeit sublinear, speedup for convex functions.

**Representative Papers:**


[3] Large Market Games with Near Optimal Efficiency (EC 2016) with R. Cole
ALEXANDROS VOUDOURIS (Homepage, CV)

**Thesis:** Design and Analysis of Algorithms for Non-Cooperative Environments

**Advisor:** Ioannis Caragiannis, University of Patras

**Brief Biography:** I was awarded my Ph.D. in September 2018 by the University of Patras, where I was advised by Ioannis Caragiannis. Since then, I have been working as a postdoctoral researcher at the Department of Computer Science, University of Oxford, under the supervision of Edith Elkind. My main research interests are on the design of simple algorithms (mechanisms) with provable efficiency guarantees for many fundamental problems in Algorithmic Game Theory and Computational Social Choice. I received my M.S.c. in Computer Science and Technology (December 2014), and my Diploma (5-year degree) in Computer Engineering and Informatics (July 2013) from the University of Patras.

**Research Summary:** My first research direction has focused on the *efficiency and complexity in strategic environments*. In this context, one of my most interesting works is about the efficiency of *allocation mechanisms* for the distribution of divisible resources [1]. This setting captures important scenarios, including the allocation of bandwidth in communication networks and cpu time in cloud computing, and as such it has been extensively studied in the related literature. My work deviates from previous papers, by making the more realistic assumption that the users have hard budget constraints, which limit the payments they can afford. This requires a new characterization of worst-case equilibria for the analysis of resource allocation mechanisms, as well as the design of new ones.

On the other hand, my second research direction has focused on the design and analysis of algorithms for *rank aggregation* with applications in *peer grading* [3] and *crowdsourcing environments* [2]. Peer grading is a method that has been adopted by the most prominent MOOC platforms to address the problem of grading the huge number of students that participate. According to a particularly simple variant of peer grading, known as *ordinal peer grading*, each student is assigned a small number of exam papers and is asked to *order* them in terms of quality from best to worst. Given the *partial* rankings provided by the students, rank aggregation methods can merge them into a global complete one, representing the relative performance of the students. Naturally, this raises many questions related to the efficiency of rank aggregation methods in terms of how well the outcome ranking agrees with the *ground truth* ranking, which a professional grader would come up with. In [3] we proposed a framework for the analysis of simple rank aggregation rules, by combining theory, simulations, as well as field experiments.

**Representative Papers:**


DAVID WAJC (Homepage, CV, Scholar)

**Thesis:** Matching Algorithms Under Uncertainty, and Matching Lower Bounds

**Advisor:** Bernhard Haeupler, Carnegie Mellon University

**Brief Biography:** David is a PhD candidate at Carnegie Mellon University's computer science department. He is broadly interested in algorithms, in particular algorithms under uncertainty, including online, dynamic and distributed algorithms. His research has been published in major theory of computation venues, including FOCS, SODA, EC, PODC and ICALP. Before joining CMU, he was a Research Engineer at Yahoo! Labs, after completing an MSc and BSc (summa cum laude) at the Technion. During his studies, David has spent a semester at EPFL, has gone on numerous visits to the Simons Theory of Computing Institute at Berkeley, and has interned at Google Research and IBM R&D.

**Research Summary:** The proliferation of user-facing mobile and web-based apps has made online problems rise in prominence. In many such applications an online algorithm must match agents (e.g., riders and passengers) immediately and irrevocably on arrival of an agent or matching opportunity. How can such an algorithm guarantee good performance (competitiveness) compared to the hindsight-optimal solution? Much of my research addresses this question.

One example of such online matching-related problems is Internet ad allocation. For this problem a worst-case optimal competitive ratio of \(1 - \frac{1}{e}\) was known, but experiments showed this problem is easier in practice. In an EC'15/TEAC'18 paper we gave a possible explanation of this behavior, by studying instances with an imbalanced thicknesses on the advertisers’ and ad slots’ sides. For such instances we showed that greedy fares well, with a competitive ratio tending to one as this imbalance grows. We then designed optimal online algorithms, whose competitive ratios tend exponentially faster to one in terms of this imbalance.

In another work (SODA'18), we studied online matching in the well-studied class of \(d\)-regular graphs, for which we presented tight \(1 - \Theta(1/\sqrt{d})\) upper and lower bounds. Underlying our work is an online rounding scheme for bounded fractional matchings, which we later used (in FOCS'19) to optimally edge color graphs online.

Finally, a beautiful algorithm of Karp et al. proved the greedy algorithm is sub-optimal for online matching in bipartite graphs under one-sided vertex arrivals. Whether there exist better-than-greedy algorithms for edge arrivals or general vertex arrivals were vexing open questions. In recent work (also in FOCS'19) we answered these questions, negatively for the former, and positively for the latter.

**Representative Papers:**

[1] Online Matching with General Arrivals (FOCS'19)  
with B. Gamlath, M. Kapralov, A. Maggiori, O. Svensson

[2] Tight Bounds for Online Edge Coloring (FOCS'19)  
with I. R. Cohen and B. Peng

[3] Randomized Online Matching in Regular Graphs (SODA'18)  
with I. R. Cohen

with J. Naor
FANG-YI YU (Homepage, CV, Scholar)

Thesis: Dynamics on Social Networks

Advisor: Grant Schoenebeck, University of Michigan

Brief Biography: I am currently a post-doctoral research fellow working with Grant Schoenebeck. I obtained my Ph.D. degree in Computer Science in August 2019 from the Computer Science and Engineering Division at the University of Michigan. I received the B.S. degree in Electrical Engineering with double major in Mathematics from the National Taiwan University in 2013.

Research Summary: The majority of my work involves studying the long-term behavior of dynamical systems with applications, including contagions and opinion formation on social networks, local search algorithms (stochastic gradient descent), and equilibria of no-regret learners [1, 2, 3, 4].

In a recent work [4], I study a large family of stochastic processes which contains stochastic approximation algorithm and stochastic gradient descent with a uniform step size. A key question is how this family of stochastic processes is approximated by their mean-field approximations. We provide a tight analysis: for any non-attracting fixed point in any stochastic process in this family, we show that system can escape the fixed point in $O(n \log n)$ time with high probability. We also show that it takes time $\Omega(n \log n)$ to escape such a fixed point with any constant probability. This result improves previous analysis of stochastic gradient descent escaping saddle points, and provide new insight on evolutionary stable strategies in evolutionary game theory.

In another work [3], I propose a family of binary opinion formation models, including several previous dynamics. I prove the tight bound on the consensus rate on the dense Erdos-Renyi random graphs when the dynamics are “majority-like.” Technically, I propose a general framework that upper bounds the hitting time of homogeneous irreversible Markov chain, which is robust against small perturbation.

Recently, I also worked on information elicitation mechanisms which incentivize agents to report their signals truthfully even in the absence of verification. I made connections to variational methods in statistics which enables new information elicitation mechanisms in the continuous setting and a deeper understanding of information elicitation.

Representative Papers:


[3] Consensus of Interacting Particle Systems on Erdos-Renyi Graphs (SODA 18) with Schoenebeck, G.

MANOLIS ZAMPETAKIS (Homepage, CV)

**Thesis:** Efficient Algorithms for Truncated and Mixture Models

**Advisor:** Constantinos Daskalakis, Massachusetts Institute of Technology

**Brief Biography:** Manolis Zampetakis is a Ph.D. student at MIT advised by Constantinos Daskalakis. His research interests include statistics, theoretical machine learning, complexity theory and mechanism design. He is a recipient of the 2018 Google PhD Fellowship on “Algorithms, Optimizations, and Market”. He has interned at Google Research, NY (2017), Yahoo! Research, NY (2018) and Microsoft Research, New England (2019). He has organized workshops on complexity of total problems (FOCS’18) and on algorithms for learning and economics (WALE 2019).

**Research Summary:** My research is focused on how the foundations of algorithms and complexity influence other fields like Statistics, Machine Learning and Economics. I believe that these fields significantly benefit from the wide toolkit of TCS and at the same time they transform TCS by introducing new concepts and posing relevant questions that lie beyond the limit of our current understanding.

**Truncated Statistics.** A classical challenge in Statistics is estimation from truncated samples. Truncation occurs when samples falling outside of a subset of the support of the distribution are not observed. Truncation has myriad manifestations in all areas of the economical and physical sciences. As a simple illustration, the values that insurance adjusters observe are truncated. Indeed, clients usually only report losses that are over their deductible. In our FOCS’18, COLT’19, FOCS’19 papers, we provide the first efficient algorithms to perform statistical estimation from truncated samples in the cases of Gaussian estimation and linear regression.

**Expectation - Maximization.** The Expectation-Maximization (EM) algorithm, from 1977, is one of the most widely used heuristics for statistical estimation under mixture models with application to medical and economical sciences. Nevertheless, little is known about its theoretical convergence, even in the paradigmatic case of mixture of two multi-normal distributions. In our COLT’17 paper we show that in this case the EM algorithm globally and efficiently converges to the true parameters.

**Complexity of Total Problems and Cryptography.** An unfulfilled goal of cryptography is to build cryptographic primitives whose security is based on the hardness of a whole complexity class. The notion of NP-hardness has been proven inadequate for this purpose and hence other complexity classes need to be explored. In our FOCS’18 paper we achieve the first necessary step in this direction by identifying the first “natural” PPP-complete problem that is related to lattice-based crypto. This way we answer a longstanding open question from the seminal paper of Papadimitriou 1994 on total search problems. Our work reveals a research direction with open problems that relate complexity theory, cryptography and game theory.

**Representative Papers:**

1. Efficient Statistics, in High Dimensions, from Truncated Samples (FOCS 2018) with C. Daskalakis, T. Gouleakis, and C. Tzamos
2. Ten Steps of EM Suffice for Mixtures of Two Gaussians (COLT 2017) with C. Daskalakis and C. Tzamos
3. PPP-completeness with Connections to Cryptography (FOCS 2018) with K. Sotiraki and G. Zindelis
Index

asynchronous optimization
   Yixin Tao, 18

blockchain
   Bo Li, 13

cloud computing
   Ludwig Dierks, 8

combinatorial auctions
   Ophir Friedler, 10

complexity theory
   Manolis Zampetakis, 22

dynamic algorithms
   David Waje, 20

dynamic auctions
   Yuan Deng, 7

dynamical systems
   Fang-Yi Yu, 21

fair division
   Biaoshuai Tao, 17
   Bo Li, 13
   Rupert Freeman, 9
   Yixin Tao, 18

inference
   Aleck Johnsen, 12

influence maximization
   Amin Rahimian, 14
   Biaoshuai Tao, 17

information elicitation
   Fang-Yi Yu, 21
   Rupert Freeman, 9

learning & data science
   Nikhil Garg, 11

machine learning
   Manolis Zampetakis, 22
   Navid Azizan, 6

market design
   Ludwig Dierks, 8
   Navid Azizan, 6
   Nikhil Garg, 11
   Ophir Friedler, 10
   market efficiency and dynamics
   Yixin Tao, 18

matching
   David Waje, 20

matching markets
   Ali Shameli, 16

mechanism design
   Aleck Johnsen, 12
   Ali Shameli, 16
   Ariel Schwartzman Cohenca, 15
   Bo Li, 13
   Manolis Zampetakis, 22
   Ophir Friedler, 10
   Yuan Deng, 7

menu complexity
   Ariel Schwartzman Cohenca, 15

networks
   Navid Azizan, 6

online algorithms
   Bo Li, 13
   David Waje, 20

online learning
   Yuan Deng, 7

online markets
   Aleck Johnsen, 12

online platforms
   Nikhil Garg, 11

operations research
   Ludwig Dierks, 8

optimization
   Navid Azizan, 6

peer grading
   Alexandros Voudouris, 19

price of anarchy
   Alexandros Voudouris, 19

pricing
   Navid Azizan, 6

prior independence
   Aleck Johnsen, 12

queuing theory
   Ludwig Dierks, 8
rank aggregation
  Alexandros Voudouris, 19
resource allocation
  Alexandros Voudouris, 19
  Biaoshuai Tao, 17
revenue management
  Ali Shameli, 16

simple auctions
  Ophir Friedler, 10
social choice
  Nikhil Garg, 11
  Rupert Freeman, 9
social contagion
  Amin Rahimian, 14
social learning
  Amin Rahimian, 14
social networks
  Amin Rahimian, 14
  Biaoshuai Tao, 17
  Fang-Yi Yu, 21
statistics
  Manolis Zampetakis, 22

tournament design
  Ariel Schwartzman Cohenca, 15
Query complexity and communication complexity of equilibria have been actively studied in the past decade. Recent progress in these fields of informational complexity has led to a quite good understanding of equilibria. This survey summarizes the established results for the three most common solution concepts: Nash equilibria, correlated equilibria, and coarse correlated equilibria. The survey provides a high-level idea of the techniques that are utilized to deduce recently developed lower bounds on Nash equilibria.

Categories and Subject Descriptors: Theory of computation [Theory and algorithms for application domains]: Algorithmic game theory and mechanism design—Exact and approximate computation of equilibria, Convergence and learning in games

General Terms: Complexity of Equilibria

Additional Key Words and Phrases: Communication complexity, Query complexity, Nash equilibrium, Correlated equilibrium, Coarse correlated equilibrium, Potential games

1. INTRODUCTION

Query complexity and communication complexity are two well-established and well-studied complexity models used to study how much information from the input are needed to find an output of a problem. Query complexity studies a setting where an algorithm initially has no information about the input but can (adaptively) ask queries about the input to find an output. Communication complexity studies a setting where the input is distributed between Alice and Bob (or between multiple parties) and Alice and Bob are allowed to communicate in order to find an output. In a game-theoretic context, let $S$ be some solution concept of a normal-form game; E.g., $S$ could be the set of all Nash equilibria, the set of all approximate Nash equilibria, the set of correlated equilibria etc. Query and communication complexity of $S$ are defined as follows.

- **Query complexity.** Initially the algorithm is ignorant of the payoffs in the game. The *query complexity of a solution concept* $S$ captures the number of payoff queries that are needed for an algorithm to find a solution in $S$.

- **Communication complexity.** Initially every player knows only his own utility function. The *communication complexity of a solution concept* $S$ captures the number of bits that players need to communicate to each other to find a solution in $S$.

We start by discussing the particular relevance of these two informational models in the game-theoretic context.
1.1 Informational Complexity and the Speed of Learning

An underlying assumption of any equilibrium notion is that players predict correctly the behavior of their opponents (or the correlated distribution over the action profiles). One justification for this problematic assumption, which appears in the seminal work of [Nash 1951], is that in some scenarios players may learn the behavior of their opponents in cases where the game is played repeatedly. This idea led to an extensive study of learning dynamics and their convergence to equilibria; see, e.g., [Young 2004; Hart and Mas-Colell 2013; Kalai and Lehrer 1993].

One natural, and general, class of adaptive dynamics is that of uncoupled dynamics [Hart and Mas-Colell 2003; 2006], where it is assumed that players do not know the utilities of their opponents but observe their past behavior. The possible and the impossible with respect to the existence of uncoupled dynamics that lead to equilibria is quite well understood. Regret minimizing dynamics are known to converge to correlated equilibria [Hart and Mas-Colell 2000; Hart 2005; Blum and Monsour 2007]. Several uncoupled dynamics that converge to approximate Nash equilibria [Foster and Young 2006; Hart and Mas-Colell 2006; Germano and Lugosi 2007; Young 2009; Babichenko 2012] have been introduced. The known dynamics that converge to Nash equilibria and to correlated equilibria have very different characteristics. While regret-minimizing dynamics converge to correlated equilibria persistently and fast (persistency is captured by the fact that the regret of a player forms a supermartingale), the dynamics that converge to Nash equilibria are based on an exhaustive search principle and the convergence is slow.

Informational complexity models and, in particular, communication complexity models constitute a formal framework for studying the rate of convergence of various dynamics. Such models do not specify exact dynamic under consideration but, instead provide bounds on the rate of convergence for all dynamics in a given class. As has been pointed out by [Conitzer and Sandholm 2004], for every solution concept (in particular equilibrium solutions), the communication complexity of a solution is identical (up to a logarithmic factor) to the rate of convergence of any uncoupled dynamics to this solution. Therefore, studying the communication complexity of an equilibrium notion is essentially equivalent to studying the rate of convergence of uncoupled dynamics to this equilibrium notion.

1.2 Informational Complexity versus Computational Complexity

The broad agenda of equilibrium complexity research is to examine whether a given equilibrium notion is an appropriate solution by understanding the hardness of computing the equilibrium. The hardness of computing equilibria in games with many players requires clarification about the input representation. Consider, for instance, $n$-player binary-action games. A utility of a single player is a mapping $u_i : \{0, 1\}^n \rightarrow \mathbb{R}$. Hence, a direct representation of a game requires $c \cdot n \cdot 2^n$ bits, when $c$ is the number of bits needed to represent a single payoff in the game.\footnote{The description of a game with a constant number of players might also be problematic if we consider games with a huge number of actions such as dueling games [Immorlica et al. 2011] and Blotto games [Hart 2008; Behnezhad et al. 2019].}

Throughout this survey we assume that $c$ is polylogarithmic in the size of the game. To study the hardness of computing equilibria in games with many players, we shall first...
address the issue that a general instance of the problem has an exponential input size.

One approach, which has been adopted in the computational complexity literature, focuses on succinctly representable classes of games such as graphical games, anonymous games, congestion games, and more. The informational complexity literature takes a different approach: it imposes informational restrictions. Query complexity studies a scenario where an algorithm (or a player in the game) is willing to compute an equilibrium in the game, but it (or he) does not know the payoffs. The algorithm (or the player) can deduce the payoff of a specific player (his own payoff or an opponent’s payoff) in a specific action profile in constant time. Communication complexity studies a scenario where each player knows only his own payoff.

There are two conceptual differences between informational and computational complexity models. First, the domain of the problem differs. Computational models apply to succinctly representable classes of games. By contrast, informational complexity models typically apply to all games without restrictions, or to non-succinctly representable classes. Moreover, since we do not impose any computational restrictions in the informational complexity models, if the game is succinctly representable we typically can communicate the entire game in a reasonable number of bits and then compute an equilibrium.\(^2\) The second difference is in the type of results. Typical hardness results in the computational model are either \(\text{PPAD}\)-hardness, \(\text{PLS}\)-hardness, \(\text{NP}\)-hardness, or that of another complexity class. Namely, hardness results in the computational model rely on an exponential hypothesis of the corresponding class. By contrast, hardness results in informational models typically do not rely on any hypothesis and provide precise bounds on the amount of information needed for achieving an equilibrium. This is a good place to mention that even though from a conceptual perspective informational complexity models differ from computational complexity models, from a technical perspective the techniques developed to solve informational complexity problems are commonly useful for tackling computational complexity problems and vice versa. Moreover, the existing literature indicates a very tight connection between the computational environment and the informational environment in a variety of settings (e.g., combinatorial allocation, submodular maximization, and many others). Whenever a problem in the computational environment admits an efficient algorithm, its analog in the informational environment typically can be solved efficiently (insofar as the amount of information is concerned). Similarly for negative results. Whenever a problem in the computational environment is shown to be hard, its analog in the informational environment typically requires a large amount of information.

The survey is organized as follows. Section 2 provides the definitions of the central notions in this survey. We summarize the known complexity results in Sections 3, 4, 5, and 6. Section 3 focuses on the most fundamental setting where normal form games are considered and the goal is to classify whether the complexity is polynomial or polylogarithmic in the size of the game. Section 4 discusses specific classes of games. Section 5 provides tighter bounds beyond the polynomial versus

\(^2\)The query model is more delicate because we require payoff queries rather than direct queries of bits from the input. See Section 4.
logarithmic scale. Section 6 discusses the dependence of the results on the approximation value. Section 7 examines which of the solution concepts are total and succinct (i.e., which solution concepts guarantee the existence of a solution that can be represented succinctly for all games). In Sections 8 and 9 we provide high-level ideas of the techniques that have been utilized to prove informational hardness of equilibria. Section 8 starts with very simple and naive techniques and discusses the limitations of these techniques. Section 9 provides the key ingredients of the (much) more involved recent techniques in this field. Finally, Section 10 concludes with some open problems.

2. DEFINITIONS

2.1 Solution Concepts

The definitions of all but one solution concept are standard. In the case of $\epsilon$-correlated equilibria in games with many actions the definition should be formulated carefully. We briefly define the standard notions and then discuss in more detail the case of $\epsilon$-correlated equilibria.

In an $n$-player $m$-action game the set of players is $[n]$. The set of actions of player $i$ is $A_i = [m]$. The set of mixed actions of player $i$ is $X_i = \Delta([m])$. The set of pure action profiles is $A = [m]^n$. The set of mixed action profiles is $X = X_1 \times \ldots \times X_n$. The set of correlated distributions is $C = \Delta(A)$. A utility of player $i$ is given by $u_i : A \rightarrow [0,1]$. This utility function can be extended to mixed action profiles and denoted by $u_i(x) = \mathbb{E}_{a \sim \Delta X_i} u_i(a_1, \ldots, a_n)$. The utility can also be extended to correlated distributions and denoted by $u_i(c) = \mathbb{E}_{a \sim \Delta A_i} u_i(a_i, x_i)$. Given $a \in A$ ($x \in X$) we denote by $a_{-i}$ ($x_{-i} \in X_{-i}$) the profile of $i$’s opponents. Given $x_{-i} \in X_{-i}$ we denote the best-reply value of player $i$ against $x_{-i}$ by $br(x_{-i}) = \max_{a_i \in A_i} u_i(a_i, x_{-i}) \in [0,1]$.

Definition 2.1. An action profile $a \in A$ is a pure Nash equilibrium if for every player $i \in [n]$ it holds that $u_i(a) \geq u_i(a', a_{-i})$ for every $a' \in A_i$.

Definition 2.2. A mixed action profile $x \in X$ is a Nash equilibrium if for every player $i \in [n]$ it holds that $u_i(x) \geq u_i(a'_i, x_{-i})$ for every $a'_i \in A_i$. Equivalently, $x \in X$ is a Nash equilibrium if for every player $i \in [n]$ it holds that $u_i(x) = br(x_{-i})$.

The first definition in 2.2 can be extended to the following approximate notion.

Definition 2.3. A mixed action profile $x \in X$ is an $\epsilon$-Nash equilibrium if for every player $i \in [n]$ it holds that $u_i(x) \geq u_i(a'_i, x_{-i}) - \epsilon$ for every $a'_i \in A_i$.

The second definition in 2.2 can be extended to the following approximate notion.

Definition 2.4. A mixed action profile $x \in X$ is an $\epsilon$-well-supported Nash equilibrium if for every player $i \in [n]$ it holds that $u_i(a_i, x_{-i}) \geq br(x_{-i}) - \epsilon$ for every action $a_i$ that is played with positive probability (i.e., $x_i(a_i) > 0$).

To define approximate correlated equilibrium we use the notion of a swapping policy, which is a function $f : [m] \rightarrow [m]$.

Definition 2.5. A correlated distribution $c \in C = \Delta(A)$ is a correlated equilibrium if for every swapping policy $f$ of player $i$ it holds that $u_i(c) \geq \mathbb{E}_{(a_i,a_{-i}) \sim c} u_i(f(a_i), a_{-i})$. 

ACM SIGecom Exchanges, Vol. 17, No. 2, October 2019, Pages 25–45
This definition of correlated equilibrium is somewhat overcomplicated, because we could restrict attention to the class of $m^2$ swapping policies $(f_{j,k})_{j,k \in [m]}$ of the form $f_{j,k}(b) = b$ if $b \neq j$ and $f_{j,k}(j) = k$ (i.e., we could swap only action $j$ for action $k$ when $j$ is recommended). It can be easily shown that for an exact correlated equilibrium the requirement $u_i(c) \geq E_{(a_i,a_{-i})}u_i(f(a_i),a_{-i})$ for every swapping policy $f$ is equivalent to the requirement $u_i(c) \geq E_{(a_i,a_{-i})}u_i(f(a_i),a_{-i})$ for every swapping policy $f \in \{f_{j,k}\}_{j,k \in [m]}$.

Definition 2.5 can be extended to the following approximate notion.

**Definition 2.6.** A correlated distribution $c \in C = \Delta(A)$ is an $\epsilon$-correlated equilibrium if for every swapping policy $f$ of player $i$ it holds that

$$u_i(c) \geq E_{(a_i,a_{-i})}u_i(f(a_i),a_{-i}) - \epsilon.$$  

We emphasize that for a constant value of $\epsilon$ and for a large number of actions, the definition that requires that $u_i(c) \geq E_{(a_i,a_{-i})}u_i(f(a_i),a_{-i}) - \epsilon$ only for simple swapping policies $f = f_{j,k}$ is meaningless. For instance, the uniform distribution over all action profiles is a $\frac{1}{m}$-correlated equilibrium for all games. Indeed, with probability $1 - \frac{1}{m}$ action $j$ will be recommended and then the utility $u_i(a_i,a_{-i})$ is identical to the utility $u_i(f(a_i),a_{-i})$.

Finally, a coarse correlated equilibrium and its approximate notion can be defined by restricting the family of constant swapping functions to constant swaps $\{f_k\}$ such that $f_k(j) \equiv k$. Formally,

**Definition 2.7.** A correlated distribution $c \in C = \Delta(A)$ is a coarse correlated equilibrium if for every player $i$ and action $a_i' \in A_i$ it holds that

$$u_i(c) \geq E_{(a_i,a_{-i})}u_i(a_i',a_{-i}).$$

**Definition 2.8.** A correlated distribution $c \in C = \Delta(A)$ is an $\epsilon$-coarse correlated equilibrium if for every player $i$ and action $a_i' \in A_i$ it holds that

$$u_i(c) \geq E_{(a_i,a_{-i})}u_i(a_i',a_{-i}) - \epsilon.$$  

The solution concepts and their approximate notions satisfy the inclusions

$$\text{PNE} \subset \text{NE} \subset \text{CE} \subset \text{CCE} \quad \epsilon\text{-PNE} \subset \epsilon\text{-NE} \subset \epsilon\text{-CE} \subset \epsilon\text{-CCE}.$$  

For the case of Nash equilibria we also have that $\text{NE} \subset \epsilon\text{-WSNE} \subset \epsilon\text{-NE}$.

### 2.2 Informational Complexity Models

Query complexity problems are defined as follows. A query protocol maps every history of past queries and answers to either an additional query or to an output. In our case queries are pairs $(a,i)$ and the answer is $u_i(a)$.

For deterministic query complexity, this mapping is deterministic, and the cost of a protocol is defined to be the maximal number of queries (across all inputs, i.e., games) until an output is produced. A protocol is correct if it outputs a correct answer (i.e., an equilibrium) for all games. The query complexity of an equilibrium is defined to be the minimal cost across all correct protocols.

For randomized query complexity the mapping from histories to queries might be random, and the cost of a protocol is defined to be the maximal expected number of queries (maximum across all inputs and the expectation is taken with respect
to the randomization of the protocol) until an output is produced. A protocol is correct if it outputs a correct answer with probability $2/3$ (again with respect to the randomization of the protocol) for all games. The query complexity of an equilibrium is defined to be the minimal cost across all correct protocols.

In communication complexity the definitions are similar. A communication protocol maps every history of communication to either a 0/1 message from Alice to Bob, a 0/1 message from Bob to Alice, or a termination. In the case of termination Alice and Bob need to produce an outcome.

For deterministic communication complexity, this mapping is deterministic, and the cost of a protocol is defined to be the maximal number of messages (across all inputs, i.e., games) until termination. A protocol is correct if the output of Alice coincides with the output of Bob and is correct (i.e., is an equilibrium) for all games. The communication complexity of an equilibrium is defined to be the minimal cost across all correct protocols.

For randomized communication complexity, we assume that Alice and Bob have access to an infinite string of random public coin flips. The mapping from histories to messages might be random (i.e., based on the realizations of the random public coin flips), and the cost of a protocol is defined to be the maximal expected number of queries (maximum across all inputs and the expectation is taken with respect to the random public coin flips) until termination. A protocol is correct with probability $2/3$ (again with respect to the randomization of the protocol) for all games. The communication complexity of an equilibrium is defined to be the minimal cost across all correct protocols.

For the case of more than two players, the definitions are similar, where in each step of the protocol player $i$ sends a public message to all the other players.

We refer the reader to the books of [Kushilevitz and Nisan 1997; Arora and Barak 2009] on the topic of communication complexity.

Note that every query protocol (deterministic or random) with cost $h$ induces a communication protocol with cost $ch$, where $c$ is the number of bits needed to represent a single payoff in the game. Instead of querying the payoff of player $i$, player $i$ sends $c$ public messages to the other players and then all players know this payoff, as in the query model. Hence the communication complexity of any problem is at most the query complexity of that problem (multiplied by a negligible factor of the representation size of a single payoff).

It is interesting to note that all negative communication complexity results presented in this survey hold not only for the $n$-party communication problem but even in the simpler problem where Alice holds as a private input the utilities of players $i = 1, \ldots, n/2$ and Bob holds the utilities of players $i = n/2 + 1, \ldots, n$. The positive results, on the other hand, apply to the more restrictive model of $n$-party communication.

3. POLYNOMIAL VERSUS LOGARITHMIC COMPLEXITY

Note that in each informational model, that of query complexity or communication complexity a trivial upper bound on the complexity of a problem is the input
size: players can just query or communicate the entire game.\(^3\) Probably the most fundamental question that can be asked is whether the complexity is polynomial or polylogarithmic in the input size. This survey focuses mainly on this question. A discussion of more precise bounds (i.e., the power of the polynomial or the logarithm) is relegated to Section 5. After almost two decades of study, we currently have a very good understanding of the informational complexity of the most common equilibrium notions. The results are summarized in Tables I and II. These tables include an additional column that specifies whether the solution concept is succinctly total. Simply speaking, a solution concept is succinctly total if every game admits a succinctly representable solution, where succinctness is defined to be polylogarithmic in the size of the game. Note that succinctness of the output is a very natural desideratum in this setting, as we simply want the outcome of the algorithm (or the communication) to be of reasonable size. For further discussion on succinct totality see Section 7.

The notion of \(\epsilon\)-equilibrium in Tables I and II refers to an additive and constant value of approximation. Other values of approximation are discussed in Section 6.

<table>
<thead>
<tr>
<th></th>
<th>Succ Tot</th>
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<th>Rand QC</th>
<th>Det CC</th>
<th>Rand CC</th>
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<td>poly</td>
<td>poly</td>
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<td>poly</td>
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<td>poly</td>
</tr>
<tr>
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<td>log</td>
<td>log</td>
<td>log</td>
</tr>
<tr>
<td>(\epsilon)-correlated</td>
<td>Yes</td>
<td>poly</td>
<td>log</td>
<td>log</td>
<td>log</td>
</tr>
</tbody>
</table>

Table I. Complexity of equilibria in \(n\)-player binary-action games. Polynomial complexity means that a \(2^{O(n)}\) lower bound is known. Logarithmic complexity means that a \(\text{poly}(n)\) upper bound is known.

<table>
<thead>
<tr>
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<th>Succ Tot</th>
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<th>Rand QC</th>
<th>Det CC</th>
<th>Rand CC</th>
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</thead>
<tbody>
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<td>poly</td>
<td>poly</td>
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</tr>
<tr>
<td>Nash</td>
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</tr>
<tr>
<td>(\epsilon)-Nash</td>
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<tr>
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<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>(\epsilon)-correlated</td>
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<td>poly</td>
<td>?</td>
<td>?</td>
<td>log</td>
</tr>
<tr>
<td>Coarse correlated</td>
<td>Yes</td>
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<td>poly</td>
</tr>
</tbody>
</table>

Table II. Complexity of equilibria in two-player \(m\)-action games. Polynomial complexity means that a \(\text{poly}(m)\) lower bound is known. Logarithmic complexity means that a \(\text{polylog}(m)\) upper bound is known.

We discuss the results of Tables I and II for each of the solution concepts in turn.

**Nash equilibria.** The results indicate that all variants of Nash equilibrium problems in all informational models require polynomial information in the size of the game. The deduction of these results has a long history, which is summarized below. The study of informational complexity of equilibria was initiated by [Conitzer

\(^3\)We impose no computational restrictions on the players, and so the computation of an equilibrium once the entire game is known comes with no cost.
and Sandholm 2004] who showed an $\Omega(m^2)$ communication complexity bound on computing a pure Nash equilibrium in two-player $m$-action games. [Conitzer and Sandholm 2004] also presented results about more basic solution concepts such as sequential elimination of dominated strategies. This hardness result is obviously valid also for the query complexity model. [Hart and Mansour 2010] proved the hardness of a pure Nash equilibrium in $n$-player games, as well as the communication hardness of an exact Nash equilibrium in $n$-player games. [Babichenko 2016] proved the first result for a total variant of a Nash equilibrium problem. Specifically, [Babichenko 2016] showed that an $\epsilon$-well-supported Nash equilibrium requires an $\exp(n)$ number of queries. Subsequently, [Chen et al. 2017] extended the result to $\epsilon$-Nash equilibrium for slightly worse bound of $\exp(\frac{n}{\log n})$. Finally, [Rubinstein 2016] improved the bound to $\exp(n)$ for an $\epsilon$-Nash equilibrium. The communication complexity lower bounds on an approximate Nash equilibrium (in the two-player and the $n$-player settings) was proved in [Babichenko and Rubinstein 2017].

**Correlated equilibria.** The informational complexity of correlated equilibrium notions is more intricate. The story starts with regret-minimizing algorithms [Littlestone and Warmuth 1994; Cesa-Bianchi and Lugosi 2006] that guarantee convergence of the regret to 0 at a rate that is polynomial (quadratic) in the approximation and logarithmic in the number of actions. [Hart and Mas-Colell 2000; 2001] showed the connection of these regret minimizing algorithms to the learning of correlated equilibrium. The regret-minimizing algorithms are translated to an algorithm whose empirical distribution of play forms an $\epsilon$-correlated equilibrium in time that is logarithmic in the number of players ($n$), polynomial in the approximation value ($\epsilon$), and (only) polynomial in the number of actions ($m$). Indeed, in the definition of an approximate correlated equilibrium (Definition 2.6), we require that the regret of the $m^m$ swapping functions be low. This requires $\log(m^{m}) = m \log m$ iterations of regret-minimizing algorithms. See the discussion after Definition 2.6 on why it is necessary to consider all the $m^m$ swapping functions to achieve an appropriate notion of approximate equilibrium.

In each step of the algorithm, the updating of the regrets requires only $nm$ payoff queries. In the communication setting the updating of the regrets requires no communication. This yields a $\text{poly}(n, m)$ query algorithm and a corresponding $\text{poly}(n, m)$ communication protocol. Note that in $n$-player games with a constant number of actions, this implies an algorithm that is logarithmic in the input (i.e., $\text{poly}(n)$) for a correlated equilibrium.\(^4\) By contrast, for two-player games with many actions, this only implies an algorithm that is polynomial in the input for a correlated equilibrium.

It is important to notice that regret-minimizing algorithms are randomized and that they converge to an approximate correlated equilibrium. [Hart and Nisan 2018] showed that in the query complexity model, the properties of randomization and of producing an approximate correlated equilibrium are both essential for solving the problem in $\text{poly}(n)$ queries. Namely, the deterministic query complexity of an

\(^4\)Moreover, in an alternative standard query model where an answer to a query is the payoff profile rather than the payoff of a specific player, only $O(\log(n))$ payoff-profile queries are sufficient; see [Goldberg and Roth 2016]. However, note that each single query reveals $\Theta(n)$ information about the game.
\( \epsilon \)-correlated equilibrium is exponential in \( n \) and the randomized query complexity of an exact correlated equilibrium is exponential in \( n \).

In contrast to query complexity, the communication complexity of a correlated equilibrium is polynomial in \( n \) even for exact correlated equilibrium, and even for deterministic communication protocols. Based on the ellipsoid against hope ideas of [Papadimitriou and Roughgarden 2008], [Jiang and Leyton-Brown 2015] showed a deterministic algorithm that computes an exact correlated equilibrium with \( \text{poly}(n) \) communication.

**Coarse correlated equilibria.** Note that we did not specify any results for coarse correlated equilibria in Table I. The reason for that is that in binary-action games a coarse correlated equilibrium (and its approximate notion) is identical to a correlated equilibrium. Hence, the same results apply. However, for two-player games with many actions, these equilibria notions differ.

Regret-minimizing algorithms converge to an approximate coarse correlated equilibrium in a rate that is logarithmic in the number of actions. Together with the observation that in the communication complexity setting players know their own regret (in particular, without the need for payoff queries), we can deduce that regret-minimizing algorithms require just \( \text{polylog}(m) \) rounds to converge to an \( \epsilon \)-coarse correlated equilibrium. Each round requires us to specify a single played action which again requires \( \log(m) \) communication.

The negative results for the query complexity of an approximate coarse correlated equilibrium (Corollary 8.1) and the communication complexity of an exact coarse correlated equilibrium (Corollary 8.2) appear later in this survey.

### 4. SPECIFIC CLASSES OF GAMES

The hardness of Nash equilibria in general games raises the question of whether it can be computed in classes of games. For this problem to be meaningful in an informational complexity setting, we need to choose a class of games that does not admit a succinct representation. Obviously, any succinctly representable game can be solved in the communication model by a protocol in which every player communicates his succinctly representable utility to the others: it requires low communication. In the query model the situation is slightly more complicated. The standard query model we have considered so far uses payoff queries. If queries are allowed to ask about input, this solves the problem. But what if queries are restricted to asking about payoffs? One can ask how the restriction to payoff queries might affect equilibrium computation in a variety of classes of games, such as congestion games [Fearnley et al. 2015], graphical games, polymatrix games, anonymous games, et cetera. Interestingly, there is a generic answer to all of these questions that has been provided in [Goldberg and Roth 2016]. [Goldberg and Roth 2016] focus on the most general formulation of succinctly representable games: a collection of \( 2^k \) games (which might have no relation to each other). [Goldberg and Roth 2016] prove the existence of a robust query protocol that computes an \( \epsilon \)-Nash equilibrium in a \( \text{poly}(k) \) number of queries. The algorithm is based on the

---

5In parallel, [Babichenko and Barman 2015] showed the weaker result that the deterministic query complexity of an exact correlated equilibrium is exponential in \( n \).
idea of having a hypothetical game in mind (in their case it is defined to be the median among all consistent games in the class), computing an equilibrium for this game (which incurs no cost in the query model), and checking whether the computed equilibrium is an approximate equilibrium of the actual game. The key point is that whenever the computed equilibrium of the hypothetical game is not an equilibrium, a large (constant) fraction of the consistent games are eliminated.

4.1 Potential Games

There are few of the fundamental classes of games in game theory that have no succinct representation. One important class is that of potential games [Monderer and Shapley 1996], which includes as a special case the class of congestion games. Potential games are characterized by the property of the existence of a single potential function that maps action profiles to real numbers. This potential function captures gains and losses from unilateral deviations by any single player; Namely, the difference between the potential values of two action profiles that is obtained by unilateral deviation is equal to the difference between the utilities of the deviating player in these two action profiles. Pure Nash equilibria in potential games are the local maxima (with respect to unilateral deviations) of the potential function and, in particular, a pure Nash equilibrium is guaranteed to exist. The query complexity of a pure Nash equilibrium in potential games has been studied by [Nisan 2009], who showed a \( \Omega(2^n) \) lower bound for the case of \( n \)-player binary-action games. Recently, [Babichenko et al. 2019] showed the hardness of a pure Nash equilibrium in the communication model. Concretely, [Babichenko et al. 2019] show a \( \text{poly}(m) \) lower bound in two-player \( m \)-action games and a \( 2^{\Omega(n)} \) lower bound for \( n \)-player binary-action games. Interestingly, the problem of finding a pure Nash equilibrium in potential games might be viewed as a succinctly total problem; see Section 7.1.

For an \( \epsilon \)-Nash equilibrium there exists an efficient communication protocol that requires only \( \frac{2n}{\epsilon} n(n + \log m) \) communication in \( n \)-player \( m \)-action games. The same protocol requires \( \frac{2n}{\epsilon} n^2 m \) queries, which is efficient for games with a constant number of actions. This protocol implements the \( \epsilon \)-better-reply dynamic, where in each step a single player updates his action to an \( \epsilon \)-better one if such an action exists. This dynamic yields an \( \epsilon \)-improvement of the potential every time a player updates an action. A single improvement requires \( n + \log m \) communication. Communication of \( n \) bits is needed to find a player that can make an improvement, and then \( \log m \) bits are needed to describe the better action. The key observation is that the potential function of a potential game with payoffs in \([0, 1]\) has a potential function is bounded in \([-n, n]\) and, therefore, the number of \( \epsilon \)-improvements is bounded by \( \frac{2n}{\epsilon} \).

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6In fact, potential games are equivalent to congestion games where the number of resources might not be polynomial in the number of players and hence their representation size is not polynomial in the number of players.

7One cannot expect to have polylogarithmic dependence on \( m \) in the query model. The hidden dominant strategy (see Section 8.1) generates a potential game.
5. TIGHTER BOUNDS

We have a better understanding of complexity for certain equilibrium notions in two-player games. Table III summarizes the known lower and upper bounds.

<table>
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<th>Rand QC</th>
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<td>(m^2)</td>
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<td>(m^2)</td>
</tr>
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<td>(m^2)</td>
<td>(m^2)</td>
<td>(m^2)</td>
</tr>
<tr>
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<td>(m^2)</td>
<td>(m^2)</td>
<td>(m^2)</td>
</tr>
<tr>
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<td>(m, m^2)</td>
<td>(?, m)</td>
<td>(?, m)</td>
</tr>
<tr>
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<td>(m, m^2)</td>
<td>(m, m^2)</td>
<td>(m, m^2)</td>
</tr>
<tr>
<td>coarse correlated</td>
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<td>(m)</td>
<td>( ?, \log^* (m))</td>
<td></td>
</tr>
<tr>
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<td>(m, m^2)</td>
<td>(m)</td>
<td>( ?, \log^* (m))</td>
<td></td>
</tr>
</tbody>
</table>

Table III. Bounds on the complexity of equilibria in two-player \(m\)-action games. A single number indicates matching of lower and upper bounds up to a logarithmic factor. Two numbers indicate differing lower and upper bounds.

The first row in Table III follows from [Conitzer and Sandholm 2004], who showed that \(\Omega(m^2)\) communication is needed to determine the existence of a pure Nash equilibrium. This result implies the same bound for the more restricted query model. The second and third rows follow from a recent result by [Göös and Rubinstein 2018], who proved that not only that polynomial communication is needed for an \(\epsilon\)-Nash equilibrium in two-player games but also that, in fact, communication of \(\Omega(m^{2-\epsilon})\) is needed for every \(\epsilon > 0\) is needed; i.e., a communication of almost the entire game. Again, this result carries over to the query model.\(^8\) The fourth row follows from the representation size of a correlated equilibrium. It is shown in [Viossat 2008; Nitzan 2005] that the set of games with a unique correlated equilibrium (which is also a Nash equilibrium) is rich enough, and contains an open ball. Using these games one can deduce games where the representation size of a correlated equilibrium requires \(\Omega(m^2)\) bits for representation. The two additional cells that have some bounds (beyond those that are presented in Corollaries 8.1 and 8.2) are the randomized communication complexity of an \(\epsilon\)-correlated equilibrium and the randomized query complexity of an \(\epsilon\)-coarse correlated equilibrium. For an \(\epsilon\)-correlated equilibrium, regret-minimizing algorithms require \(O(\log(m^m)) = O(m \log(m))\) steps to converge to an equilibrium (because the number of swapping functions is \(m^m\)). Each step requires only \(\log m\) communication. For an \(\epsilon\)-coarse correlated equilibrium, it has been observed by [Goldberg and Roth 2016] that regret-minimizing algorithms require \(O(m \log m)\) queries.

For \(n\)-player games no bounds beyond the logarithmic versus polynomial are known, except for the case of a pure Nash equilibrium where it is known that \(\Omega(2^n)\) communication is needed; see [Hart and Mansour 2010].

6. THE DEPENDENCE OF THE BOUNDS ON THE APPROXIMATION VALUE

This survey have focused on approximate notions of equilibria with a small and constant value of approximation \(\epsilon\). One may ask how the complexity of the problem

\(^8\)Prior to this excellent work, no better bound than \(\Omega(n)\) was known even for the simpler query model.
depends on $\epsilon$. There are to way to approach this problem. The first is to start with problems that are tractable (i.e., of logarithmic complexity) for constant $\epsilon$ and ask what is the affect or reducing $\epsilon$ to $\epsilon = 1/\text{poly}(n)$ for $n$-player games or to $\epsilon = 1/\text{poly}(m)$ for two-player $m$-action games. The second approach is to start with problems that are intractable (i.e., of polynomial complexity) and ask how much should we increase $\epsilon$ to make the problem tractable.

We focus first on the latter approach. The positive results for $n$-player binary-action games (see Table II) remain true not only for constant $\epsilon$ but also for $\epsilon = 1/\text{poly}(n)$. This follows from the polynomial dependence on the approximation value of the regret-minimizing algorithms and of the ellipsoid against hop algorithm. For two-player $m$-action games, setting $\epsilon = 1/\text{poly}(m)$ makes all the problems intractable, including even the simplest problem of an $\epsilon$-coarse correlated equilibrium in a randomized communication complexity model. One can obtain this hardness result on $\epsilon$-coarse correlated equilibrium by analyzing the hide-and-seek game with bad actions (see Section 8.2) for a $1/\text{poly}(m)$-coarse correlated equilibrium. See also [Ganor and Karthik 2018].

Regarding the second approach of increasing $\epsilon$ and finding a sufficiently large value that makes the problem tractable, the overall picture is that we do not have a good understanding of the values of $\epsilon$ where the transition from tractable to intractable occurs. The gap between the positive and the negative results is huge. The current negative results are theoretical in nature. The deduction of these results does not involve trying to optimize the arguments with respect to $\epsilon$ and, therefore, results in bounds of order $\epsilon \approx 10^{-6}$ or in many cases even much smaller. Little is known about the positive results that significantly improve upon trivial protocols. The best values for which communicationally efficient algorithms are known are $\epsilon = 0.38$ for the Nash equilibrium and $\epsilon = 0.65$ for the well-supported-Nash equilibrium [Goldberg and Pastink 2014; Czumaj et al. 2019].

### 7. SUCCINCT TOTALITY OF EQUILIBRIA

The existence of an (approximate) equilibrium in every game is guaranteed for all solution concepts (Nash, correlated, and coarse correlated equilibrium) is guaranteed except for pure Nash equilibrium. However, the existence of a succinct equilibrium is not guaranteed for all the (approximate) solution concepts. In informational complexity problems, it is desirable that the output be of much smaller size than the input (i.e., polylogarithmic). We briefly discuss which of the above solution concepts are guaranteed to have a succinctly representable solution.

**Which problems are not total?** All the exact solution concepts in two-player $m$-action games are not total. The description of an exact Nash equilibrium requires us to specify the probability distribution of $m$ actions, which requires a polynomial description in the size of the game. Exact correlated and coarse correlated equilibria requires to specify a distribution over the $m^2$ profiles. It is easy to construct games (e.g., zero-sum games) where all correlated (coarse correlated) equilibria

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9One example of a trivial protocol might be: Alice sends to Bob the action profile $(a, b)$ of her best outcome in the game, and then Bob plays with equal probability the action $b$ and the action $b^*$ that is the best reply to $a$. This protocol leads to a $\frac{1}{2}$-Nash equilibrium.
have support $\Omega(m)$ and hence the description of such an equilibrium is $\text{poly}(m)$.

An exact mixed Nash equilibrium in $n$-player binary-action games requires us to specify only $n$ real numbers in $[0,1]$, which in principle looks to be succinct. However, [Hart and Mansour 2010] showed that even for games with outcomes in $\{0,1,2\}$, the description of an exact Nash equilibrium might require a doubly-exponential precision of the mixed strategy. Hence this problem is not total.

**Which problems are total?** Correlated equilibria in $n$-player binary-action games are given by a linear program with $2^n$ variables but only $2n$ constraints. Hence the existence of a correlated equilibrium (an extreme point of the feasible set) with support $2n$ is guaranteed.

The existence of an $\epsilon$-Nash equilibrium whose mixed strategies are located on a grid of size $\Omega(\frac{1}{n})$ is guaranteed by a simple rounding of the probabilities to the grid points.$^{10}$

In two-player $m$-action games, [Lipton et al. 2003] showed that the existence of $\epsilon$-Nash equilibrium where both players randomize uniformly over a multi-set of $O(\log(m))$ actions. Such a strategy requires only $O(\log(m)^2)$ bits of representation. This $\epsilon$-Nash equilibrium is, in particular, an $\epsilon$-correlated and an $\epsilon$-coarse correlated equilibrium whose support is succinct over the action profiles.$^{11}$

**Non-deterministic communication complexity.** Simply speaking, the non-deterministic complexity of a problem counts the output size of the problem and the amount of communication needed to verify the solution. Note that verification of equilibria requires a single bit of communication: a player who knows his own utility function can verify whether he has a better response (or an $\epsilon$-better response) and send this information to the other players. Hence, the non-deterministic communication complexity for all the succinctly total equilibrium notions is low (logarithmic in the input).

### 7.1 Potential Games

We recall that a problem is succinctly total if it has a succinct solution for every input. Potential games have a restriction on the input: the game needs to have the potential structure. Therefore, the problem of finding a pure Nash equilibrium in potential games is prima facie not total. However, by combining the following two observations:

(a) There is succinct evidence that a game is not a potential game. This evidence comes in the form of a unilaterally deviating cycle where the sum of gains and losses from the deviations is not equal to 0; see [Monderer and Shapley 1996]. A unilaterally deviating cycle is a cycle of profiles of length at most $2n$ that is

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$^{10}$More involved arguments can show the existence of an $\epsilon$-Nash equilibrium on a grid of size $\Omega(\frac{1}{\log(m)})$; see [Babichenko et al. 2016]. The existence of an $\epsilon$-Nash equilibrium on a grid of size $\Omega(1)$ remains an open question.

$^{11}$It is interesting to notice that standard linear programming arguments fail to prove the existence of an $\epsilon$-correlated equilibrium with support $\text{polylog}(m)$. From a mathematical point of view, this means that we use Brouwer’s fixed-point theorem to prove the existence of a sparsely supported correlated equilibrium. I am not aware of a proof that avoids the use of Brouwer’s fixed-point theorem.
obtained by unilateral deviations.

(b) There is a communicationally efficient protocol to verify whether a game is a potential game; otherwise, it outputs a succinct evidence of unilaterally deviating cycle; see [Babichenko et al. 2019].

we can define a close variant of the pure Nash equilibrium problem: the algorithm should either output a pure Nash equilibrium or it should output a unilaterally deviating cycle where gains and losses do not sum up to 0. This problem is total, of low non-deterministic complexity, and is communicationally hard; see [Babichenko et al. 2019]. Very recently, [Babichenko and Rubinstein 2019] extended the hardness of finding a Nash equilibrium in potential games from the case of a pure Nash equilibrium to the general case of any (possibly mixed) equilibrium.

8. SIMPLE TECHNIQUES

There are several very simple techniques to “hide” equilibria in games. I find it useful to start the technical discussion with these simple techniques and to understand their limitations (i.e., in which environments they do not work). An advantage of these simple techniques is that in cases where they succeed in providing a lower bound, this lower bound applies to the weakest solution concept of coarse correlated equilibrium.

8.1 A Hidden Dominant Strategy

In a two-player game (and in fact even in a single-player decision problem), we can choose uniformly at random a single action $a^* \in [m]$ of Player 1, and set his utilities to be $u_1(a_1, a_2) = 1_{a_1=a^*}$. In the query model, Player 1 will need $O(m)$ queries (even in the randomized model) to find this hidden strategy. This observation shows that even for the weakest solution concept, namely, the $\epsilon$-coarse correlated equilibrium, the query complexity is polynomial in the input.

Corollary 8.1. The randomized query complexity of an $\epsilon$-coarse correlated equilibrium in two-player $m$-action games is $\Omega(m)$.

Note that in the communication complexity setting this technique fails: Player 1 knows his entire utility function and hence identifies the dominant strategy immediately. The following example applies to the communication complexity setting.

8.2 Hide-and-Seek Game with Bad Actions of the Hider

Consider a hide-and-seek zero-sum game with $u_2(a_1, a_2) = -u_1(a_1, a_2) = 1_{a_1=a_2}$ where Player 1 is the Hider and Player 2 is the Seeker. It is easy to check that any exact coarse-correlated equilibrium has uniform marginals for both players.

We next slightly modify this game by making half of the Hider’s actions dominated. We pick a subset $B \subset [m]$ with $|B| = m/2$ uniformly at random, and we set $u_1(a_1, a_2) = -2 \cdot 1_{a_1 \not\in B} - 1_{a_1=a_2}$. We do not change the utilities of the Seeker, which remain $u_2(a_1, a_2) = 1_{a_1=a_2}$. Simple arguments of elimination of dominated strategies show that in any coarse correlated equilibrium the marginals for both players are the uniform distribution over $B$. Note that the Seeker has no information about $B$ from his utility, but a coarse correlated equilibrium identifies $B$. 

ACM SIGecom Exchanges, Vol. 17, No. 2, October 2019, Pages 25–45
Namely, in any communication protocol the Hider has to communicate $B$ to the Seeker, which requires $O(m)$ communication.

**Corollary 8.2.** The randomized communication complexity of an exact correlated equilibrium in two-player $m$-action games is $\Omega(m)$.

Note that this construction fails to provide bounds for approximate notions of equilibria. The following approximate Nash equilibrium requires no communication: the Hider chooses a location uniformly at random from $B$ and the Seeker chooses a location uniformly at random from $[m]$. By best-replying to the Hider’s strategy, the Seeker can increase his expected payoff from $1/m$ to $2/m$. Hence this profile of mixed actions forms a $1/m$-Nash equilibrium.\(^{12}\)

## 9. RECENT LOWER BOUND TECHNIQUES FOR NASH EQUILIBRIA

Several recent papers have succeeded in proving lower bounds on Nash equilibria in the hard-to-prove environment where the problem turns out to be total (and of low non-deterministic communication complexity); see [Babichenko 2016; Babichenko and Rubinstein 2017; Göös and Rubinstein 2018; Babichenko et al. 2019; Babichenko and Rubinstein 2019]. Even though the results are in different settings (query or communication complexity, two-player or $n$-player games, general or potential games), all the proofs share a common structure. This common structure consists of the following six ingredients which are discussed in more details thereafter.

1. **Start with a query-hard end-of-line problem.**
2. **In the communication model, “lift” the query-hard problem to a communicationally hard end-of-line problem.**
3. **Embed the line in the end-of-line problem as a continuous function.**
4. **Embed the function as a continuous-action imitation game.**
5. **In the communication model, introduce into the imitation game incentives to report truthfully the private local information about the line.**
6. **Discretize of the imitation game.**

### 1. Query-hard end-of-line problem.

The starting point of the above listed reductions is some end of (a single) line problem over a low-degree graph. The starting point of the line is known and the task is to find its end. This low-degree graph might be directed or undirected, and the line can be metered or unmetered depending on the application.\(^{13}\) **Local behavior of a line** in a given vertex is specified by whether the line passes through this vertex and, if so, what are its previous visit and its next visit. It is crucial that the underlying graph will have a low degree (in

\(^{12}\)The same technique can be applied to prove the communicational hardness of an $\epsilon$-Nash equilibrium for $\epsilon = \text{poly}(1/m)$, but not beyond that.

\(^{13}\)A line is metered if the vertex through which the line passes indicates on the distance of the line from its origin. For instance, a line over a positively directed two-dimensional grid is metered because the pair of coordinates $(x, y)$ indicate that the line has passed through $x + y$ vertices so far.
fact, in many proofs it has a constant degree) because then the local behavior of the line will have a very succinct (constant) representation.

2. Communicationally hard end-of-line problem. For communication complexity problems, the reduction should start with some communicationally hard problem. The tool that is utilized here is simulation theorems. Simulation theorems are a beautiful tool of recent development; see [Raz and McKenzie 1999; Göös and Pitassi 2014; Göös et al. 2015; 2017], just to mention a few. The idea is to “lift” a query-hard problem to a communicatively hard problem by carefully splitting the information about the problem between Alice and Bob. An outstanding fact about simulation theorems that they can be applied to any problem. The typical gadget that is used to carefully distribute information is the index gadget: for each element in the input Alice holds an array of possible inputs and Bob holds an index of the correct element.\(^{14}\) Note that this distribution of information is very different from the one we have in game-theoretic settings, where each player simply knows his own utility function; therefore additional (and quite substantial) work is needed in order to apply this beautiful tool to game-theoretic problems.

To summarize, the second step in the established proof techniques (based on existing literature on simulation theorems) defines a communicationally hard instance of end-of-line where Alice holds arrays that contain information on possible local behavior of the line in each vertex and Bob holds indices on where the correct local behavior of the line is hidden in Alice’s arrays.

3. Embedding a line as a continuous function. In the next step it will become clearer why we insist on “making the end-of-line problem continuous”; for now, we will just describe what it means. We embed our host graph in some well structured graph as the \(\delta\)-grid of \([0,1]^k\). We embed the line (that currently is over the \(\delta\)-grid) to a Lipschitz function \(f : [0,1]^k \to [0,1]^k\) (or, in some applications to a potential function \(f : [0,1]^k \to [0,1]\) ). The key properties that we want from this construction are locality and reducibility.

Locality means that the value of the function at a point \(x \in [0,1]^k\) can be calculated from the local behavior of the line in the neighborhood of \(x\) when, roughly speaking, the local behavior of the line with respect to a continuous point \(x \in [0,1]^k\) is defined by the local behavior at the closest grid point.

Reducibility means that all solutions of \(f\) should be located close to the end of the line, where solutions are interpreted as fixed points in the case of \(f : [0,1]^k \to [0,1]^k\) and are interpreted as local maxima in the case of a potential function \(f : [0,1]^k \to [0,1]\).

Such a (highly non-trivial) embedding was introduced in [Hirsch et al. 1989] for the case of \(f : [0,1]^k \to [0,1]^k\). Latter modifications of this construction appear in [Rubinstein 2016; Chen and Deng 2008]. In the case of a potential function, variants of such constructions appear in [Hubáček and Yogev 2017; Babichenko and Rubinstein 2019].

\(^{14}\)In some applications (e.g., [Göös and Rubinstein 2018; Babichenko et al. 2019]) it is crucial that the size of these arrays will be constant. A simulation theorem for an index gadget of constant size is known only for specific problems. Luckily for us, one of these problems is a variant of an end-of-line problem; see [Göös and Rubinstein 2018].
This is a good place to mention the closely related literature on the query and communication complexity of finding a fixed point.\textsuperscript{15} In the fixed point problem the input is a continuous function $f : A \to A$, where $A$ is a compact convex set and the output is an $\epsilon$-fixed point of the function $f$. In the query model [Hirsch et al. 1989] show an exponential, in the dimension, lower bound for this query problem (i.e., an $\exp(n)$ lower bound for the case of $A = [0, 1]^n$), even for an $\epsilon$-fixed point with a constant $\epsilon$. [Hirsch et al. 1989] show also a polynomial in $\frac{1}{\epsilon}$ lower bound for the two-dimensional problem (i.e., a $\poly(\frac{1}{\epsilon})$ lower bound for the case of $A = [0, 1]^2$). Recently, communication variants of this problem have been studied. For instance, in the decomposition problem Alice holds a function $f : A \to A$, Bob holds a function $g : A \to A$, and their goal is to compute a fixed point of the decomposition $f \circ g$. [Ganor et al. 2019; Roughgarden and Weinstein 2016] show that communication variants of fixed point computation are as hard as the query problem; namely, they show that the lower bounds of [Hirsch et al. 1989] apply to the communication problem as well.

4. Embedding the function as a continuous-action imitation game. The main obstacle in proving hardness results on mixed equilibrium notions comes from the fact that it is typically hard to prove the nonexistence of malicious equilibria (with large support). To demonstrate this point we suggest some naive (and not completely specified) approach to solve end-of-line with an equilibrium of some game.

For sake of simplicity, assume that the given line passes through all the vertices of $G$. We propose the following simple game: Alice and Bob choose a vertex. We can design a game whose incentives reflect that Alice wants to be the successor of Bob, and Bob wants to match Alice. Indeed if we focus on pure Nash equilibria the only stable scenario is the the profile where Alice and Bob choose the end-of-line vertex where both are happy (the end of the line is defined to be the successor of itself). However, once we focus on approximate Nash equilibria the action profile where both players are randomizing uniformly is an approximate equilibrium. Indeed, Alice would gain only $\frac{1}{|V|}$ by deviating to the end-of-line vertex. This equilibrium does not provide any information about the location of the end of the line, an obstacle common to many such naive approaches.

However, one can overcome this obstacle when the problem is continuous. Given a continuous function $f : [0, 1]^k \to [0, 1]^k$ there is a very simple two-player imitation game with a continuum of actions all of whose Nash equilibria are pure and correspond to fixed points of $f$: Alice and Bob choose points $x, y \in [0, 1]^k$. Alice wants to match $f(y)$ and Bob wants to match $x$. More specifically, their utilities are given by $u_A(x, y) = -\|x - f(y)\|_2^2$ and $u_B(x, y) = -\|y - x\|_2^2$. Even if Bob is playing a mixed strategy $\beta \in \Delta([0, 1]^k)$, Alice has a unique best response, which is $x^* = \mathbb{E}_{y \sim \beta}[f(y)]$. This simply follows from the fact that expectation is the unique minimizer of the square error. Similarly, Bob has a unique best reply. Therefore, any Nash equilibrium in this game is pure. Once we understand that, it is immediate to verify that the pure Nash equilibrium must be a fixed point. This idea of an

\textsuperscript{15}Indeed, the close connection between Brouwer’s fixed-point Theorem and Nash equilibria is well known in both directions [Nash 1951; Shmaya 2012].
imitation game was introduced in [McLennan and Tourky 2005; Shmaya 2012]. In some applications, more complicated imitation techniques are required (see, e.g., [Babichenko and Rubinstein 2019]).

5. Incentivizing truthful reporting. In the communication model, neither Alice nor Bob knows the line. In other words, they do not know \( f \), and hence Alice’s utility cannot be defined simply by \( -||x - f(y)||^2 \). However, for any point \( x \) Alice knows her local information at point \( x \) (see step 2). We provide Alice with a strong incentive to report this information truthfully. The same holds for Bob with respect to his point \( y \). The key point is that if Alice and Bob are choosing points close to each other, they can combine this information to deduce the local information about the line, and by the locality of \( f \) they can compute \( f(y) \) (in fact, only Alice needs to perform this computation). In other words, Alice’s utility is defined with respect to the combined reported information rather than with respect to the actual \( f \).

6. Discretization. Finally, to conclude the reduction it is necessary to the convert the action space \([0, 1]^k\) back to discrete. The elegant consequences of the purity of Nash equilibria in step 4 translates to the observation that in every approximate well-supported Nash equilibrium Alice and Bob choose actions in a small cube of the \( \delta \)-grid, in which the merging of the local information remains possible.

To strengthen the reduction to approximate Nash equilibria rather than to approximate well-supported Nash equilibria, additional techniques are needed. [Chen et al. 2017] have suggested the technique of replicating players, and [Rubinstein 2016] have suggested the technique of error-correcting codes.

10. OPEN PROBLEMS

(1) Arguably the most fundamental questions that remain open are those of logarithmic versus polynomial complexity. As indicated in Table II, most of the problems regarding the communication complexity of approximate correlated (or coarse correlated) equilibria in two-player games remain open. Specifically, for an \( \epsilon \)-correlated equilibrium in the deterministic and the randomized communication settings, is there a polynomial lower bound? Is there a polylogarithmic protocol? And do they exist for \( \epsilon \)-coarse correlated equilibrium in the deterministic communication model?

(2) For an \( \epsilon \)-Nash equilibrium in \( n \)-player binary-action games no algorithm that improves upon \( 2^n \) (in the query or communication complexity model) is known. The negative results, on the other hand, prove exponential hardness for \( (1+\delta)^n \) for a constant but very small \( \delta \). It will be interesting to close these gaps (in both the query and communication models). As indicated in Table III, there are also gaps in the polynomial complexity of correlated notions of equilibria in two-player games.

(3) As mentioned in Section 6, the complexity of an \( \epsilon \)-Nash equilibrium as a function
of constant values of $\epsilon$ is far from being understood.\(^{16}\)

(4) As indicated in Table III, there are several instances of correlated equilibria in two-player $m$-action games where the answer is known to be bounded in between $m$ and $m^2$, but the correct power of $m$ is not known.

(5) Beside potential games there is another interesting class of not succinctly representable games for which the existence of a pure approximate Nash equilibrium is guaranteed. Due to non-succinctness, informational complexity questions are relevant. [Azrieli and Shmaya 2013] introduced a class of $\lambda$-Lipschitz games where the influence of player $i$ on the utility of player $j$ is bounded by $\lambda$. They show that in every $n$-player $\lambda$-Lipschitz game with $\lambda = \tilde{O}(n^{-2})$, existence of a pure approximate Nash equilibrium is guaranteed. The complexity (informational or computational) of finding such an approximate pure Nash equilibrium is not known.

(6) Quantum computation allows us to:

(a) Define new solution concepts as quantum correlated equilibria that lie in between Nash equilibria and correlated equilibria; see [Deckelbaum 2014].
(b) Look at more powerful communication environments such as quantum communication complexity; see [Brassard 2001].

Little is known about the complexity of these quantum solution concepts, and many problems remain open in regard to quantum complexity models in game-theoretic settings. The reader is referred to [Rubinstein 2018] for a discussion of these issues.

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Multi-dimensional Mechanism Design via Random Order Contention Resolution Schemes

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Already in 1981 Myerson gave a characterization of the optimal mechanism for a single parameter Bayesian mechanism design. However, till today we have no idea for how such a characterization for the multi-dimensional setting could even look like. Moreover, it wasn’t until not that long time ago that we could not develop mechanisms for such setting with any reasonable and provable performance guarantees. The seminal work of [Chawla et al. 2009] on sequential posted pricing mechanisms gave us an approach for approximately solving the Bayesian multi-parameter unit-demand mechanism design problem (BMUMD). The paper left open the question on how to obtain a constant approximation for the matroid setting. Two mathematically beautiful results from combinatorial optimization under uncertainty where devised in order to answer this question. First, Kleinberg and Weinberg in 2011 extended the classical Prophet Inequality result into the matroid setting to give a 2-approximation for BMUMD for a single matroid setting. Second, Feldman, Svensson and Zenklusen in 2016 adapted the Contention Resolution Scheme framework for online settings. We add to this line of work by considering the Contention Resolution Scheme framework in the random order setting. The most impressive implication of this research are the new algorithms for BMUMD which improve the previous results in the multimatroid setting. Although the range of implications of the CR Scheme framework in the random order is reasonably wide, we shall focus in this letter on presenting only the single matroid setting and how it is connected to BMUMD.

Categories and Subject Descriptors: F.2.2 [ANALYSIS OF ALGORITHMS AND PROBLEM COMPLEXITY]: Nonnumerical Algorithms and Problems—Computations on discrete structures

General Terms: Algorithms, Theory, Economics

Additional Key Words and Phrases: contention resolution schemes, stochastic optimization, approximation algorithms

1. INTRODUCTION

Contention resolution schemes have proven to be an incredibly powerful concept which allows to tackle a broad class of problems. The framework has been initially designed to handle submodular optimization under various types of constraints, that is, intersections of matroids (and more generally exchange systems), knapsacks, and unsplittable flows on trees. Later on, it turned out that this framework perfectly extends to optimization under uncertainty, like stochastic probing and online selection problems, which further can be applied to mechanism design.

We add to this line of work by showing how to create contention resolution
schemes for intersection of matroids and knapsacks when we work in the random order setting. More precisely, we do know the whole universe of elements in advance, but they appear in an order given by a random permutation. Upon arrival we need to irrevocably decide whether to take an element or not. The main application of our framework is a $k + 4 + \varepsilon$ approximation ratio for the Bayesian multi-parameter unit-demand mechanism design under the constraint of $k$ matroids intersection, which improves upon the previous bounds of $4k - 2$ and $e(k + 1)$.

Contention resolution schemes

Let us start with an illustrative problem. Consider a matroid $M = (E, I)$ and a fractional solution $x$ from its polytope. Suppose we are given a weight vector $w : E \mapsto \mathbb{R}_+$, and we look for an algorithm that returns an independent set $S \in I$ such that \( \sum_{e \in S} w_e \geq c \cdot \sum_{e \in E} w_e x_e \) for some constant $c < 1$. The idea is to settle for a randomized algorithm and demand that every element is taken into $S$ with probability at least $c \cdot x_e$. Such a property would immediately entail the desired guarantee.

How to design an algorithm returning $S$ such that $\mathbb{P}[e \in S] \geq c \cdot x_e$? Chekuri et al. [Chekuri et al. 2014] presented a framework of contention resolution schemes (CR schemes) which address this problem, among other applications. The idea is to first draw a random set $R(x)$ such that $\mathbb{P}[e \in R(x)] = x_e$ for each $e \in E$ independently, and afterwards – since $R(x)$ is most likely not an independent set in $I$ – to drop some elements from $R(x)$ to meet the feasibility constraint, that is, to resolve the contention between the elements.

Our contribution. Simply speaking, we show that the above problem can be solved also if we work in a random order model, i.e., when elements of $E$ appear to us according to a uniformly random permutation, and upon arrival we need to make an irrevocable decision of whether to take an element or not.

In its full generality Chekuri et al. were dealing not only with matroids but arbitrary intersections of matroids, knapsacks, exchange systems, and unsplittable flow on trees. They were also maximizing not only linear functions, but non-negative submodular functions as well. We do so as well, restricted to intersections of matroid and knapsack constraints. For a single matroid and a linear objective, Chekuri et al. obtained an approximation (the constant $c$) of $1 - \frac{1}{e}$, while we get $\frac{1}{2}$. However, for intersection of $k$ matroids, starting with $k \geq 2$, we obtain a better bound of $\frac{1}{1 + \frac{1}{k}}$, improving upon theirs $\frac{1}{e(k + o(k))}$, even though we work in a more restrictive model. The following is the most important implication of our framework.

**Theorem 1.1.** There exists a random-order CR scheme for intersection of $k$ matroids with $c = \frac{1}{1 + \frac{1}{k}}$.

And thanks to it we can obtain the result for the BMUMD problem.

Bayesian multi-parameter unit-demand mechanism design

Consider the following mechanism design problem. There are $n$ agents and a single seller providing a set of services. The agent $i$ is interested in buying the $i$-th service and values its as $v_i$, which is drawn independently from a distribution $D_i$. Such a setting is called single-parameter. The valuation $v_i$ is private, but the distribu-
tion $D_i$ is known in advance. The seller can provide only a subset of services, that belongs to a system $I \in 2^{[n]}$, which is specified by feasibility constraints. A mechanism accepts bids of agents, decides on subset of agents to serve, and sets individual prices for the service. A mechanism is called truthful if agents are motivated to bid their true valuations. Myerson’s theory of virtual valuations yields truthful mechanisms that maximize the expected revenue of a seller [Myerson 1981], although they sometimes might be impractical [Ausubel and Milgrom 2006]. On the other hand, practical mechanisms are often non-truthful [Ausubel and Milgrom 2006]. The Sequential Posted Pricing Mechanism (SPM) introduced by Chawla et al. [Chawla et al. 2010] gives a nice trade-off – it is truthful, simple to implement, and gives near-optimal revenue. An SPM offers each agent a ‘take-it-or-leave-it’ price for a service. After refusal the service shall not be provided, so it is easy to see that an SPM is indeed a truthful mechanism.

The paragraph above concerns only the single-parameter setup. In the Bayesian multi-parameter unit-demand mechanism design (BMUMD for short), we have $n$ buyers and one seller. The seller offers a number of different services indexed by set $J$. The set $J$ is partitioned into groups $J_i$, with the services in $J_i$ being targeted by agent $i$. Each agent $i$ is interested in getting any one of the services in $J_i$, i.e., agents are unit-demand. Agent $i$ has value $v_j$ for service $j \in J_i$. Value $v_j$ is independent of all other values and is drawn from distribution $D_j$. Once again the seller faces a feasibility constraint specified by a set system $I \subseteq 2^J$.

Unlike single-parameter setup, this problem is not solvable efficiently by the well-established Myerson’s approach. The paper of Chawla et al. [Chawla et al. 2010] launched a line of work in obtaining approximate results for the multi-parameter setup, by suggesting a possible avenue of a solution via the so-called Oblivious Posted Price mechanisms. One would have to first embed the multi-parameter problem into a single-parameter one, and later to ensure that the algorithm would work if the items are presented in an adversary order. Kleinberg and Weinberg [Kleinberg and Weinberg 2012] solved the BMUMD problem for matroid environments with approximation of $4k - 2$ for intersection of $k$ matroids (with 2-approximation for a single matroid), but they have not used the Oblivious Posted Price mechanisms. Feldman et al. [Feldman et al. 2016] devised the first Oblivious Posted Price mechanisms and obtained an $ek + o(k)$ approximation for the intersection of $k$ matroids.

Our contribution. We observe that the Oblivious Posted Price is an overly demanding notion, and we need to handle the oblivious order only when looking at the items of a given client, but there is no need to restrict the order of clients. In our algorithm we randomly shuffle clients, but cannot make assumption on the client’s choice. This hybrid approach is what allows us to obtain improved bounds. For $k = 2$ we match up to $\varepsilon$ the 6-approximation of Kleinberg and Weinberg [Kleinberg and Weinberg 2012], but starting from $k \geq 3$ our ratios are better; for $k = 3$ we get $7 + \varepsilon$ improving over 9.48 of Feldman et al. [Feldman et al. 2016].

**Theorem 1.2.** Bayesian multi-parameter unit-demand mechanism design over $k$ matroid constraints admits a $(k + 4 + \varepsilon)$ approximation for any $\varepsilon > 0$.
2. CONTENTION RESOLUTION SCHEME FOR A SINGLE MATROID

Consider a uniform matroid \( M = (E, \mathcal{I} \subseteq \binom{E}{k}) \) of rank \( k \). We consider optimization over the matroid polytope \( P(M) = \{ x \in \mathbb{R}^E_{\geq 0} \mid \sum_{e \in E} x_e \leq k \} \).

We shall use the following two properties. Both facts hold for any matroid.

**Fact 2.1.** We can represent any \( x \in P(M) \) as \( x = \sum_{i=1}^m \beta_i \cdot 1_{B_i} \), where \( B_1, \ldots, B_m \in \mathcal{I} \) and \( \beta_1, \ldots, \beta_m \) are non-negative weights such that \( \sum_{i=1}^m \beta_i = 1 \) in \( P(M) \).

The following holds for any matroid, although in the case of a uniform matroid this statement is almost trivial. It is a delicate generalization of the Basis Exchange lemma [Schrijver 2003].

**Fact 2.2.** Let \( A, B \in \mathcal{I} \) be two independent sets of matroid \( M = (E, \mathcal{I} \subseteq \binom{E}{k}) \). We can find an assignment \( \phi[A, B] : A \mapsto B \cup \{\perp\} \) such that:

1. \( \phi[A, B](e) = e \) for every \( e \in A \cap B \),
2. for each \( f \in B \) there exists at most one \( e \in A \) for which \( \phi[A, B](e) = f \),
3. for \( e \in A \setminus B \), if \( \phi[A, B](e) = \perp \), then \( B + e \in \mathcal{I} \), otherwise \( B - \phi[A, B](e) - e \in \mathcal{I} \).

The procedure. The procedure is shown in the Algorithm 1. What shall be important in the analysis of the approximation is that we do not assume that we know the whole set \( R(x) \) in advance, but rather we reveal the set \( R(x) \) after each step in we get to know what elements belongs to \( R(x) \) only in line 7. We do it just to ease the analysis by using the principle of deferred decisions.

Remember that the input of the algorithm is a point \( x \in P(M) \) and a random set \( R(x) \). The algorithm’s idea is quite simple.

1. We start with the convex decomposition of \( x \) into \( \sum \beta_i \cdot B_i \).
2. In each step we take a random element \(e\) from elements we did not consider yet. If the element is not blocked yet (whatever it means at the moment), we take it into the solution \(S\) which we gradually construct.

3. We go over each set \(B_i\) of the convex decomposition, and we insert \(e\) into \(B_i\) while removing some other element \(f\) from \(B_i\). We do it according to the mapping from Fact 2.2, which ensures that after every such swap \(B_i\) is still an independent set. The mapping has to be calculated from scratch at every iteration. This yields the correctness of the procedure, because the solution \(S\) always belongs to each \(B_i\), which means that \(S\) is independent as well.

However, we have to say something about how we block elements, because this is what affects the approximation guarantee. For this, given an element \(e\) we consider all sets \(B_i\) to which \(e\) belongs. Since \(\sum_{i:e \in B_i} \beta_i = x_e\), we choose randomly exactly one \(B_i\) according to probability \(\frac{\beta_i}{x_e}\). We call this set a critical set and denote it by \(C_e\).

Now the blocking event is the removal of \(e\) from \(C_e\). If such an event happens, then we shall discard the element, call it blocked, and not take it into the solution.

The whole procedure is presented in Algorithm 1.

**Algorithm 1 Random order contention resolution scheme for a single matroid**

1: Given: matroid \(M\), \(x \in P[M]\), and a random set \(R(x)\) such that \(P[e \in R(x)] = x_e\) for each \(e \in E\) independently
2: decompose \(x\) into \(\sum \beta_i \cdot B_i\)
3: for each element \(e\) choose a set \(B_i : e \in B_i\) with probability \(\frac{\beta_i}{x_e}\); call it a critical set, and denote it by \(C_e\)
4: \(S \leftarrow \emptyset\)
5: for each \(e \in E\) in a random order
6: \hspace{1cm} if \(e \notin R(x)\) then
7: \hspace{2cm} continue
8: \hspace{1cm} if \(e\) is not blocked (i.e., still \(e \in C_e\)) then
9: \hspace{2cm} \(S \leftarrow S \cup \{e\}\)
10: \hspace{1cm} for each set \(B_i\) such that \(e \notin B_i\) do
11: \hspace{2cm} \(B_i \leftarrow B_i + e - \phi[C_e, B_i](e)\)
12: \hspace{1cm} return \(S\)

*Approximation guarantee.* Suppose we are after \(t\) steps. Consider an element \(e\). Suppose that 1) \(e \in R(x)\), and 2) suppose that it is still not blocked. If so, then the probability that we shall take \(e\) into the solution \(S\) in step \(t + 1\) is just equal to the probability of picking \(e\) among all remaining elements, and this is just \(\frac{1}{n - t}\).

What is the probability that we shall block \(e\) (meaning \(e\) will be removed from its critical set \(C_e\))? Let us look more precisely at the probability that a given set \(B_i\) will cause a removal of \(e\) from \(C_e\). For a given \(B_i\) there exists at most one \(f \in B_i\) such that \(\phi[B_i, C_e](f) = e\). This \(f\) (if it exists) in step \(t + 1\):

- is chosen with probability \(\frac{1}{n - t}\).
it belongs to $R(x)$ with probability $x_f$,
– it has chosen $B_i$ as its critical set with probability $\beta_i/x_f$.

Hence the probability that $B_i$ is the cause of removing $e$ from $C_e$ is at most $\frac{1}{n-t}x_f \cdot \frac{\beta_i}{x_f}$. Summing over all $B_i$ we get that the probability of removing $e$ from $C_e$ in step $t+1$ is at most $\sum_{i:e \notin B_i} \frac{\beta_i}{n-t} \leq \frac{1}{n-t}$. Hence

$$P[e \text{ blocked in step } t+1] \leq \frac{1}{n-t} = P[e \text{ taken in step } t+1].$$

This one step inequality implies a global inequality saying that at the end of the whole process $P[e \text{ blocked}] \leq P[e \text{ taken}]$ (although this needs a bit more formalization using the martingale framework, which we shall omit).

Since the fate of $e$ is either to be taken or to be blocked, one of the two has to happen at some point, i.e., $P[e \text{ taken}] + P[e \text{ blocked}] = 1$. And so this finally implies that that $P[e \text{ taken}] \geq \frac{1}{2}$.

3. MULTI-PARAMETER MECHANISM DESIGN

Recall that each client $i \in \mathcal{I}$ is interested in purchasing one service from $\mathcal{J}_i$ and their valuation of an item $c \in \mathcal{J}_i$ is modeled by a random variable $v_c$, independent of other valuations, with a known distribution $D_c$. One can think that the distribution $D_c$ is always discrete.

**Bounding by auction with copies.** Imagine a setting where for each item $c \in \mathcal{J}_i$ we create an independent copy-client $c'$ interested solely in this item. The new instance has the same constraint system as the original one plus additional partition matroid. We rely on the crucial lemma from [Chawla et al. 2010], saying that the optimal revenue in the new instance can be only greater because the competition increases.

This observation allows us to obtain an LP upperbound for the true OPT. The linear program BMUMD-LP [Gupta and Nagarajan 2013] models the auction with copy-clients, which is single-parameter. $C$ denotes the set of copy-clients, which is equivalent to the set of items, and $\mathcal{P}$ is the polytope of the constraint system.

$$\max \left\{ \sum_{c \in C} \sum_p x_{c,p} \cdot p \cdot P[v_c \geq p] \right\} \quad \text{(BMUMD-LP)}$$

s.t. \(\sum_p x_{c,p} \cdot P[v_c \geq p] \in \mathcal{P}\)

\[ \sum_p x_{c,p} \leq 1 \quad \forall c \in C \]

\[ \sum_{c \in \mathcal{J}_i} \sum_p x_{c,p} \cdot P[v_c \geq p] \leq 1 \quad \forall i \in \mathcal{I}. \]

**Theorem 3.1.** The optimal value of BMUMD-LP is an upper bound for the maximal revenue in the multi-parameter auction.

To give a grip with the previous result: the object $\left(\sum_p x_{c,p} \cdot P[v_c \geq p]\right)_{c \in C} \in \mathcal{P}$ is now the vector which we decompose into a convex combination of independent sets.

**Single client menu.** The algorithm scans clients in random order, and presents a price menu to each client, from which the client picks one item which gives him the highest utility, or resigns from choosing if all utilities are negative. Such a procedure
clearly yields a truthful mechanism. Of course the menu presents only the items which are still not blocked at the moment of approaching a client.

We omit the details of how the subroutine which constructs the menu is implemented, but we shall state the most important property of it. Suppose that item $c$ is still not blocked when we approach the client and we want a guarantee on the probability that $c$ will be sold at price $p$. Then for any $\varepsilon > 0$ we can construct a menu with the following property (for all $p, c$).

$$\frac{1}{4} \cdot x_{c,p} \cdot P[v_c \geq p] \leq P[\text{client takes item } c \text{ at price } p] \leq \left( \frac{1}{4} + \varepsilon \right) x_{c,p} \cdot P[v_c \geq p]$$

The upperbound with $(1/4 + \varepsilon)$ in the second inequality is crucial to obtain the approximation ratio $k + 4 + \varepsilon$ stated in Theorem 1.2. Without the subroutine we would only have an upperbound of $x_{c,p} \cdot P[v_c \geq p]$, which would follow easily from the way the algorithm works, but it would be not enough for our needs.

**Algorithm.** With the subroutine to handle a single client, we are ready to state our algorithm for the BMUMD problem.

### Algorithm 2 Auction mechanism

1. solve the BMUMD-LP
2. for each client $i \in I$ in random order do
3. construct a menu from the non-blocked items in $J_i$
4. if client $i$ chooses $c$ then
5. go over all sets of the decomposition and update them according to item $c$

### 4. OPEN PROBLEM

The results we obtained allowed for a Contention Resolution Scheme on $k$ matroids such that $P[e \text{ in the solution}] \geq \frac{1}{k+1} \cdot x_e$. What can be interesting is that it improved the results in an offline setting, even though it works in a random order. Given the potential of the approach, a natural question appears thus — can we apply the same combinatorial insight that uses the exchange mappings to construct Online Contention Resolution Schemes that were considered by [Feldman et al. 2016]. An interesting result would be to obtain the same guarantee of $\frac{1}{k+1} x_e$ in this more restricted setup. Such a result could be used to construct a Matroid Prophet Inequality as introduced by [Kleinberg and Weinberg 2012]. With a guarantee of $\frac{1}{k+1} x_e$ it would match their result for a single matroid, yielding a 2-approximation. However, for $k \geq 3$ it would already improve over their $4k - 2$ and $e \cdot k$ of [Feldman et al. 2016].

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Tight Revenue Gaps among Simple and Optimal Mechanisms

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Consider a fundamental problem in microeconomics: selling a single item to a number of potential buyers, who independently draw their values from regular and publicly known distributions. There are four mechanisms widely studied in the literature and widely used in practice: Myerson Auction (OPT), Sequential Posted Pricing (SPM), Second-Price Auction with Anonymous Reserve (AR), and Anonymous Pricing (AP).

OPT is revenue-optimal but complicated, which also experiences several practical issues such as fairness. AP is the simplest mechanism, but also generates the lowest revenue among these four mechanisms. SPM and AR are of intermediate complexity and revenue. A quantitative approach to comparing the relative power of these mechanisms is to study their revenue gaps, each of which is defined as the largest ratio between the revenues from a pair of mechanisms. This letter surveys some recent developments on establishing tight revenue gaps, and highlights some open questions.

Categories and Subject Descriptors: Theory of Computation: Algorithmic Mechanism Design

General Terms: Economics, Theory

Additional Key Words and Phrases: Revenue Maximization, Approximation Ratio

1. INTRODUCTION

How to maximize the expected revenue of a seller, who wants to sell an indivisible item to a number of buyers, is a central problem in microeconomics. The simplest mechanism is Anonymous Pricing (denoted by AP). Such a mechanism simply posts a price of $p \in \mathbb{R}_{\geq 0}$ to all buyers, and the item is sold out iff at least one buyer
has a value no less than this price. If the seller knows the value distributions of the buyers, he can leverage a proper price to maximize the revenue (among this family of mechanisms). Although widely-used, this is not the revenue-optimal selling method – the optimal mechanism is the prominent Myerson Auction (denoted by OPT) [Myerson 1981]. In comparison, OPT is far more complex than AP, due to two reasons:

(a) It discriminates different buyers with different value priors. Conceivably, this may incur some fairness issues, and is not feasible in some markets.

(b) It is an auction scheme instead of a pricing scheme, thus requiring more seller-to-buyer communication. This may also raise privacy concerns to the buyers, since they need to report their private values rather than make take-it-or-leave-it decisions.

These complications and other undesirable features hinder the prevalence of Myerson Auction. To address these issues, two mechanisms with intermediate complexities (compared to OPT and AP) are widely studied in the literature, and are widely adopted in practice: (a) to avoid the price discrimination, the seller can use Second-Price Auction with Anonymous Reserve (denoted by AR) [Hartline and Roughgarden 2009]; and (b) to reduce communication, the seller can employ Sequential Posted Pricing (denoted by SPM) [Chawla et al. 2010].

These four mechanisms together form the lattice structure in Figure 1, in terms of both revenue-dominance and complexity. It is well known that there is a revenue gap between any pair of mechanisms. Naturally, the reader might query that how large these gaps can be.

Indeed, quantitative analysis of these gaps is also a striking theme in algorithmic economics. To this end, the notion of approximation ratio (originated from the TCS community) turns out to be a powerful language. There is a rich literature on studying the revenue gaps/approximation ratios among various mechanisms [Bulow and Klemperer 1994; Goldberg et al. 2001; Bar-Yossef et al. 2002; Guruswami et al. 2005; Koutsoupias and Pierrakos 2013; Chen et al. 2014; 2015; Fu et al. 2015; Dütting et al. 2016; Alaei et al. 2015; Correa et al. 2017].

For the above four mechanisms, we establish in [Jin et al. 2019; Jin et al. 2019] three tight bounds and an improved bound in the canonical setting with asymmet-
Tight Revenue Gaps among Simple and Optimal Mechanisms

ric and regular distributions. Prior to our work, no tight revenue gap between any pair of the four mechanisms was known.

**SPM vs. AP.** This comparison measures the power of discrimination in pricing schemes. We establish the tight ratio of constant $C^* \approx 2.62$. Prior to our work, tight ratios were known to be (a) $n$ in the asymmetric general setting [Alaei et al. 2015]; (b) $\frac{e}{2} \approx 1.58$ in the i.i.d. regular setting; and (c) 2 in the i.i.d. general setting [Hartline 2013; Dütting et al. 2016]. Actually, we can also get the last two ratios by combining the results in [Myerson 1981; Krengel and Sucheston 1978; Hill et al. 1982], which was first observed by [Hajiaghayi et al. 2007].

**OPT vs. AP.** The same tight ratio of $C^* \approx 2.62$ is achieved by the revenue gap between SPM and AP. Indeed, in our worst case instance of the SPM vs. AP problem, the optimal Sequential Posted Pricing mechanism coincides with Myerson Auction. This result answers a central question in the “simple versus optimal” research program. Previously, an upper bound of $e \approx 2.72$ was proved by [Alaei et al. 2015].

**AR vs. AP.** This comparison concerns the relative power between auction scheme and pricing scheme, when no price discrimination is allowed. We first (a) prove an upper bound of $\frac{\pi}{6} \approx 1.64$ in the asymmetric general setting, and then (b) construct matching lower-bound instances respectively in the asymmetric regular setting and the i.i.d. general setting. Before our work, (c) in the i.i.d. regular setting, where AR is identical to OPT, an upper-bound of $\frac{e}{e-1} \approx 1.58$ was first obtained by [Chawla et al. 2010], and then was shown to be tight by [Hartline 2013].

**OPT vs. AR.** This comparison studies the power of price discrimination in auction schemes. Prior to our work, the tight ratios were known in all settings [Myerson 1981; Hartline 2013; Alaei et al. 2015] except for the asymmetric regular setting. [Hartline and Roughgarden 2009] first tackled the problem in this setting – they (a) proved an upper-bound of 4, and (b) provided a 2-approximation lower-bound instance. Although this lower bound of 2 has never been broken (for a decade) and is widely believed to be the final answer, we establish a sharper 2.15-approximation instance in [Jin et al. 2019].

More concretely, we settle all of the SPM vs. AP problem, the OPT vs. AP problem and the AR vs. AP problem by formulating each revenue gap as the objective function of a math program. This methodology was initiated by [Chen et al. 2014] and [Alaei et al. 2015]. Employing a similar approach, [Birmpas et al. 2017] recently obtained a tight price of anarchy for multi-unit auctions. Our work further supports the power of this framework in proving tight bounds. En route, we have developed an abundance of tools to handle these math programs, which may find extra applications in the future.

In the remainder of this letter, we shall focus on the SPM vs. AP problem, elaborating on how to formulate this optimization problem and then reduce it to a

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1. Throughout the letter, asymmetric distributions refer to the setting where different buyers may have distinct value distributions, as opposed to identical value distributions.
2. Here, the general setting refers to the case where the distributions are not necessarily regular.
clear math program. For the other claimed results, we invite the readers to read the full papers for more details.

2. PRELIMINARIES

We will consider the single-item Bayesian mechanism design environment, where \( n \in \mathbb{N}_{\geq 1} \) buyers independently draw their values \( \mathbf{b} = \{b_i\}_{i=1}^n \in \mathbb{R}_{\geq 0}^n \) from known distributions \( \mathbf{F} = \{F_i\}_{i=1}^n \). We begin with some necessary definitions and notations.

2.1 Value Distribution

**Regular Distribution.** For any CDF \( F \) and the corresponding PDF \( f \):

(a) The *virtual value function* is defined as \( \varphi(p) \overset{\text{def}}{=} p - \frac{1-F(p)}{f(p)} \).

(b) The *revenue-quantile curve* is defined as \( R(q) \overset{\text{def}}{=} q \cdot F^{-1}(1-q) \).

The distribution \( F \) is regular iff the virtual value function \( \varphi \) is non-decreasing, or equivalently, iff the revenue-quantile curve \( R \) is a concave function. We will work more with the second definition, since it is more convenient for our use. Denote by \( \text{Reg} \) the family of all regular distributions.

**Triangular Distribution.** This family of distributions is introduced in [Alaei et al. 2015], named in view of the shapes of their revenue-quantile curves (as Figure 2 illustrates). With two parameters \( v_i \in \mathbb{R}_{\geq 0} \) and \( q_i \in [0, 1] \), a triangular distribution \( \text{Tri}(v_i, q_i) \) has a CDF of \( F_i(p) = (1-q_i) \cdot p \cdot (1-q_i) + v_i \cdot q_i \) when \( p \in [0, v_i) \) and \( F_i(p) = 1 \) when \( p \in [v_i, \infty) \). This distribution family \( \text{Tri} \) in some sense lies on the boundary of the regular distribution family \( \text{Reg} \), and plays an important role in our results.

Particularly, when \( H \to \infty \), the distribution \( \text{Tri}(H, \frac{1}{H}) \) has a limitation CDF of \( F(p) = \frac{p}{p+1} \), for all \( p \in \mathbb{R}_{\geq 0} \). Denote this special limitation distribution by \( \text{Tri}(\infty) \), which will be involved in the worst-case instance of the SPM vs. AP problem.

2.2 Mechanisms

**Anonymous Pricing (AP).** The seller posts a price of \( p \in \mathbb{R}_{\geq 0} \) to all buyers, and the item is sold out iff at least one buyer values the item no less than this price. For brevity, we denote by \( \text{AP}(p) \overset{\text{def}}{=} p \cdot (1 - \prod_{i=1}^n F_i(p)) \) the resulting revenue.

**Sequential Posted-Pricing (SPM).** Such a mechanism involves a price vector \( p = \{p_i\}_{i=1}^n \in \mathbb{R}_{\geq 0}^n \), with each price \( p_i \) posted sequentially to the \( i \)-th coming buyer. Without loss of generality, we assume the \( i \)-th coming buyer is exactly the index-\( i \) buyer. Then, the first coming buyer with a value of \( b_i \geq p_i \) will win the item. We denote by \( \text{SPM}(p) \) the resulting revenue.

3. SEQUENTIAL POSTED PRICING VS. ANONYMOUS PRICING

We interpret this SPM vs. AP problem as the following math program, and safely drop constraint (C1) on interval \( p \in [0, 1] \) as it trivially holds.

\[
\max_{F \in \text{Reg}^n} \quad \text{SPM} = \max_{p \in \mathbb{R}_{\geq 0}^n} \{\text{SPM}(p)\} \\
\text{subject to:} \quad \text{AP}(p) = p \cdot (1 - \prod_{i=1}^n F_i(p)) \leq 1 \quad \forall p \in (1, \infty) \quad (C1)
\]
Now, consider a certain regular instance \( F = \{ F_i \}_{i=1}^n \) and an optimal Sequential Posted Pricing mechanism for it. Let \( \mathbf{p}^* = \{ p_i^* \}_{i=1}^n \) be the involved posted prices. For each buyer \( i \in [n] \), define \( v_i \overset{\text{def}}{=} p_i^* \) and \( q_i \overset{\text{def}}{=} 1 - F_i(p_i^*) \). These two parameters are crucial, in the sense that the revenue from the concerning mechanism (using the prices \( \mathbf{p}^* = \{ v_i \}_{i=1}^n \)) is fully captured by the parameters \( \{ q_i \}_{i=1}^n \). In other words, if another instance \( F = \{ F_i \}_{i=1}^n \) satisfies that \( F_i(v_i) = F_i(v_i) \) for each \( i \in [n] \), then adopting the same prices \( \mathbf{p}^* = \{ v_i \}_{i=1}^n \) to this new instance results in the same amount of expected revenue. Indeed, the revenue is given by

\[
\text{SPM}(\mathbf{p}^*) = \text{SPM}(\{ v_i \}_{i=1}^n) = \sum_{i=1}^n v_i q_i \cdot \prod_{j=1}^{i-1}(1 - q_j).
\]

As Figure 3 illustrates, let us squeeze each distribution \( F_i \) to the triangular distribution \( \text{Tri}(v_i, q_i) \). Based on the above arguments, the objective function of our math program (namely the SPM revenue) remains the same, while the constraint gets relaxed – due to the stochastic dominance \( F_i \succeq \text{Tri}(v_i, q_i) \) for each \( i \in [n] \). As mentioned, the triangle distributions lie on the boundary of the regular distribution family \( \text{Reg} \), which means we cannot squeeze the instance any further.

In addition, we can verify that any instance admits an optimal Sequential Posted Pricing mechanism \( \text{SPM}(\mathbf{p}^*) \) that uses decreasing prices \( p_1^* \geq p_2^* \geq \cdots \geq p_n^* \). This allows us to concentrate merely on those triangle instances \( \{ \text{Tri}(v_i, q_i) \}_{i=1}^n \) with \( v_1 \geq v_2 \geq \cdots \geq v_n \). Since such an instance has explicit CDF’s, plugging these CDF’s into the above \( \text{AP}(\mathbf{p}) \) and SPM revenue formulas and rearranging the constraint, we can deduce a clearer math program as follows:

![Fig. 2. Demonstration for the triangular distribution Tri(v_i, q_i).](image)
max \{\text{Tri}(v_i, q_i)\}_{i=1}^n \quad \text{SPM} = \sum_{i=1}^n v_i q_i \cdot \prod_{j=1}^{i-1} (1 - q_j) \quad (P2)
subject to: \sum_{i: v_i \geq p \ln \left(1 + \frac{v_i q_i}{1 - q_i} \cdot \frac{1}{p} \right)} - \ln(1 - p^{-1}) \quad \forall p \in (1, \infty) \quad (C2)
\quad v_1 \geq v_2 \geq \cdots \geq v_n

The next step is to show the existence of two special buyers \text{Tri}(\infty) and \text{Tri}(1, 1) in a worst case instance. Intuitively, \text{Tri}(\infty) is the most effective buyer in extracting SPM revenue – he gives an expected revenue of \lim_{H \to \infty} H \cdot \left(1 - \frac{1}{H+1}\right) = 1 but with a negligible winning probability of \lim_{H \to \infty} \frac{1}{H+1} = 0. In addition, the other special buyer \text{Tri}(1, 1) has a deterministic value of 1 – the seller can acquire a promised revenue of 1 from him, even if all the other buyers refuse their posted-price offers. For ease of presentation, we omit those technical details here.

To conclude, we write down the following math program, and solve it optimally. Notably, the optimal solution (i.e., the worst-case instance) turns out to be reached when constraint (C3) is tight everywhere for any price \(p \in (1, \infty)\).

max \{\text{Tri}(v_i, q_i)\}_{i=1}^n \quad \text{SPM} = 2 + \sum_{i=1}^n (v_i - 1) q_i \cdot \prod_{j=1}^{i-1} (1 - q_j) \quad (P3)
subject to: \sum_{i: v_i \geq p \ln \left(1 + \frac{v_i q_i}{1 - q_i} \cdot \frac{1}{p} \right)} - \ln(1 - p^{-2}) \quad \forall p \in (1, \infty) \quad (C3)
\quad v_1 > v_2 > \cdots > v_n > 1
4. OPEN QUESTIONS

This letter reports our recent works on studying the revenue gaps among different families of single-item mechanisms, mostly in the setting with asymmetric regular distributions. Despite the exciting progress, two important revenue gaps about the concerning mechanisms have yet to be understood.

OPT vs. AR. A most attractive research direction is to pin down the exact ratio of Second-Price Auction with Anonymous Reserve against Myerson Auction. Our result certifies that the final answer sits in the range of $[2.15, 2.62]$. On the other hand, we highly believe that neither of the upper and lower bounds are tight.

Interestingly, the previous lower-bound instance of [Hartline and Roughgarden 2009] consists of two buyers, yet our sharper instances in [Jin et al. 2019] involve three or four buyers. Arguably, when there are more buyers, the ratio may further increase. We thus conjecture that the worst-case instance contains an infinite number of buyers, and the optimization techniques from [Alaei et al. 2015; Jin et al. 2019; Jin et al. 2019] might be the right tool to attack it.

OPT vs. SPM. Due to the reduction in [Hajiaghayi et al. 2007; Chawla et al. 2010; Correa et al. 2019], designing the optimal Sequential Posted Pricing mechanisms is equivalent to finding the optimal stopping rules against a prophet (in a certain n-choose-1 game). Namely, the OPT vs. SPM problem has the same tight ratio as the ordered prophet inequality. For a full survey on that topic, the interested readers can turn to [Lucier 2017; Correa et al. 2018].

Other Comparisons. In the literature, some other practical mechanisms have been investigated as well, mostly measuring their revenue gaps against Myerson Auction. E.g., [Beyhaghi et al. 2018; Ma and Sivan 2019] studied Second-Price Auction with Personalized Reserves, and obtained lower and upper bounds of $[1.29, 1.50]$. Apart from that ratio, the revenue gap between this family of mechanisms and SPM or AR or AP is also interesting to explore.

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The sample complexity of learning Myerson’s optimal auction from i.i.d. samples of bidders’ values has received much attention since its introduction by Cole and Roughgarden (STOC 2014). This letter gives a brief introduction of a recent work that settles the sample complexity by showing matching upper and lower bounds, up to a poly-logarithmic factor, for all families of value distributions that have been considered in the literature. The upper bounds are unified under a novel framework, which builds on the strong revenue monotonicity by Devanur, Huang, and Psomas (STOC 2016), and an information theoretic argument. This is fundamentally different from the previous approaches that rely on either constructing an $\epsilon$-net of the mechanism space, either explicitly, or implicitly via statistical learning theory, or learning an approximately accurate version of the virtual values. To our knowledge, it is the first time information theoretical arguments are used to show sample complexity upper bounds, instead of lower bounds. The lower bounds are also unified under a meta construction of hard instances.

Categories and Subject Descriptors: J.4 [Social and Behavioral Sciences]: Economics
General Terms: Algorithms, Economics, Theory
Additional Key Words and Phrases: Sample Complexity, Revenue Maximization, Myerson Auction

1. INTRODUCTION

Alice is a college student majoring in Economics. In an effort to raise fund for her road trip in the upcoming summer, she decides to sell her old smart phone on the internet. After an hour of research, Alice finds two options. The first one is eBay, which supports an auction format that is essentially the second price auction with a reserve. Having collected the bids from different bidders, it gives the phone to the bidder with the highest bid so long as it is at least the reserve price, and charges a price that is either the reserve price or the second highest bid, whichever is higher. The second option is Yabe, a new startup platform that supports arbitrary auction formats. As a student in Economics, Alice feels obliged to take the second option and to put the theory that she learns into practice.

Alice recalls that the optimal auction by [Myerson 1981] would maximize the
revenue, if she knows who the bidders are and if further the distributions from which their values for the phone are drawn are explicitly given. In a nutshell, the optimal auction works as follows. For each bidder $i$, suppose $F_i$ and $f_i$ are the cumulative distribution function (cdf) and probability density function (pdf) of her value distributions, and $v_i$ is the realized value. Then, it maps $v_i$ to a virtual value:

$$
\phi_i(v_i) = v_i - \frac{1 - F_i(v_i)}{f_i(v_i)},
$$

and gives the phone to the bidder with the highest nonnegative virtual value: the winner pays the minimum bid that would still make her win.

**Bayesian Model.** More formally, suppose there is a single type of item for sale. Let there be $n$ bidders. Each bidder $i$ has a value for an item, $v_i \geq 0$, that is independently drawn from a Bayesian prior $D_i$. The prior $\bar{D} = D_1 \times D_2 \times \cdots \times D_n$ is publicly known to the seller and all bidders; the realization of value $v_i$, however, is private information of bidder $i$.

For simplicity, most parts of this letter consider the single-item setting, where the seller has only one copy of the item and, therefore, can allocate to at most one bidder. Nonetheless, we will briefly explain at the end of the letter how the techniques and results generalize to problems with multiple copies of the item, and even more generally to the matroid setting.

Digging deeper into Yabe, Alice finds that it does provide certain information about the bidders, but not in the forms of explicitly given priors from which their values are drawn, let alone the cdf’s and pdf’s. Instead, the types of the bidders, e.g., according to the demographic information, browsing history, etc., and certain data about each type of bidders, e.g., past bids by the same type of bidders for second-hand smart phones, market research, etc., are given by Yabe. How can Alice design an auction based on the data such that the revenue is close to the optimal revenue by Myerson’s auction?

**Sample Complexity Model.** Under an idealized assumption that the data are independent and identically distributed (i.i.d.) samples from the prior, it becomes the sample complexity model by [Cole and Roughgarden 2014]. Let $A$ be an algorithm that takes i.i.d. samples as input and outputs a truthful auction. For any family of distributions, e.g., those that are regular, the minimum number of samples such that the auction returned by algorithm $A$ is a $(1-\epsilon)$-approximation in revenue, so long as the prior comes from the family, is called the sample complexity of the algorithm, with respect to the family of distributions. The minimum sample complexity achievable by any algorithm is called the sample complexity of the family of distributions.

Of late, the sample complexity of revenue maximizing auctions has received much attention in the Algorithmic Game Theory community. This letter presents a brief
introduction of a recent result of ours in [Guo et al. 2019] that pins down the sample complexity of essentially all families of distributions that have been considered in the literature, under a unifying algorithm and analysis framework.

2. EXISTING APPROACHES AND OBSTACLES

We first present a brief overview on the previous approaches for analyzing the sample complexity of revenue maximization, which can be categorized into two groups, and explain their limitations.

Statistical Learning Theory. The first approach relies on constructing an $\epsilon$-net of the mechanism space, namely, a subset of mechanisms such that for any distribution in the family, there always exists an approximately optimal mechanism in the subset. Then, it remains to identify such an approximately optimal mechanism in the $\epsilon$-net. This can be done via a standard combination of concentration plus union bounds. Informally, the resulting sample complexity will be:

$$\frac{\log \text{ (size of the } \epsilon \text{-net)}}{\epsilon^2}.$$

The construction of the $\epsilon$-net can be either explicit (e.g., [Devanur et al. 2016], [Gonczarowski and Nisan 2017], [Gonczarowski and Weinberg 2018]), or implicit via various learning dimensions from statistical learning theory (e.g., [Morgenstern and Roughgarden 2015], [Syrgkanis 2017]).

The main limitation of this approach is that the size of the $\epsilon$-net seems to have an unavoidable exponential dependence in $\epsilon^{-1}$ (see below for an example). Recall that the sample complexity upper bound will be $\log \text{ (size of the } \epsilon \text{-net})/\epsilon^2$, this exponential dependence leads to an at least cubic dependence in $\epsilon^{-1}$ in the sample complexity upper bounds. For example, we sketch below an explicit construction of the $\epsilon$-net by [Devanur et al. 2016]. With an appropriate discretization, it suffices to consider $\epsilon^{-1}$ distinct values. Further, since the optimal auction chooses the winner to maximize virtual value, it suffices to know the ordering of 0 and $\phi_i(v)$'s for all $n$ bidders and all $\epsilon^{-1}$ values. Hence, the number of auctions that we need to consider is no more than the number of orderings over the $n\epsilon^{-1}$ virtual values $\phi_i(v)$'s and 0, which equals $(n\epsilon^{-1} + 1)!$ and is singly exponential in both $n$ and $\epsilon^{-1}$. Getting rid of the exponential dependence in $\epsilon^{-1}$ intuitively means that it suffices to consider a constant number of distinct values, which seems implausible.

Learning the Virtual Values. An alternative approach (e.g., [Cole and Roughgarden 2014], [Roughgarden and Schrijvers 2016]) is to learn the individual value distributions well enough to obtain enough approximately accurate information about the virtual values, which induces a mechanism. Then, we analyze the revenue approximation using the connections between expected revenue and virtual values. Importantly, this approach does not need to take a union bound over exponentially many candidate mechanisms, circumventing the bottleneck that introduces the undesirable cubic dependence in $\epsilon^{-1}$ in the learning theory approach. Indeed, for the

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2This form relies on the assumption that $O(\epsilon^{-2})$ samples are sufficient for estimating the expected revenue of a mechanism up to an $\epsilon$ error, which need not be true in general especially with unbounded value distributions.
Table I. Best known sample complexity bounds prior to [Guo et al. 2019]

<table>
<thead>
<tr>
<th>Setting</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
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<tbody>
<tr>
<td>Regular</td>
<td>$\Omega(\max{\sqrt{n}e^{-1}, \epsilon^{-3}})$ \textsuperscript{ad}</td>
<td>$\tilde{O}(n\epsilon^{-4})$ \textsuperscript{b}</td>
</tr>
<tr>
<td>MHR</td>
<td>$\Omega(\max{\sqrt{n/e}, \epsilon^{-3/2}})$ \textsuperscript{ad}</td>
<td>$\tilde{O}(n\epsilon^{-3})$ \textsuperscript{bc}</td>
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<tr>
<td>$[1, H]$</td>
<td>$\Omega(H\epsilon^{-2})$ \textsuperscript{d}</td>
<td>$\tilde{O}(nH\epsilon^{-3})$ \textsuperscript{b}</td>
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<tr>
<td>$[0, 1]$-additive</td>
<td>$\Omega(\epsilon^{-2})$ \textsuperscript{d}</td>
<td>$\tilde{O}(n\epsilon^{-3})$ \textsuperscript{bc}</td>
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\textsuperscript{a} [Cole and Roughgarden 2014] \textsuperscript{b} [Devanur et al. 2016] \textsuperscript{c} [Gonczarowski and Nisan 2017] \textsuperscript{d} [Huang et al. 2015] \textsuperscript{e} [Morgenstern and Roughgarden 2015]

special case of independently and identically distributed (i.i.d.) bidders with $[0, 1]$-bounded distributions and additive approximation. [Roughgarden and Schrijvers 2016] showed a sample complexity upper bound of $\tilde{O}(n^2\epsilon^{-2})$, which is the only previous example, to our knowledge, with a sub-cubic dependence in $\epsilon^{-1}$.

The main limitation of this approach roots in the form of the virtual value as defined in Eqn. (1). It involves three components, the value $v$, the complementary cumulative distribution function $1 - F_i(v)$, a.k.a., the quantile, and the pdf $f_i(v)$. Here, the value $v$ is given as input; the quantile $1 - F_i(v)$ is relatively easy to estimate accurately via standard concentration inequalities. It is, however, impossible to get an accurate estimation of the density function $f_i$ in general. As a result, it is infeasible to learn the virtual values accurately point-wise. This is a major technical hurdle that prevents existing works using this approach from getting tight sample complexity upper bounds; in particular, they all have super-linear dependence in $n$.

Even for the special case of i.i.d. bidders, the bound is quadratic in $n$ [Roughgarden and Schrijvers 2016]; the dependence is at least $n^7$ for the general case [Cole and Roughgarden 2014]. Note that a linear dependence in $n$ follows almost trivially from the learning theory approach (e.g., [Devanur et al. 2016]).

Prior Knowledge of the Distribution Family. Another limitation of the existing approaches is that they generally rely on knowing the family of distributions up-front. Even for the special case of a single bidder, the best known algorithms are different for regular, MHR, and bound-support distributions (e.g., [Huang et al. 2015]). For MHR distributions, we may simply pick the optimal price w.r.t. the empirical distribution, i.e., the uniform distribution over the samples. For regular and $[1, H]$ bounded support distributions, however, we need to introduce a threshold $\delta > 0$ and to choose the optimal price subject to having a sale probability at least $\delta$. Further, the threshold is chosen differently for regular and $[1, H]$ bounded support distributions. If we fail to introduce a threshold when it is an arbitrary regular distribution, the expected revenue may not converge to the optimal at all [Dhangwatnotai et al. 2015]. If we set the threshold under the belief that the distribution has a $[1, H]$ bounded support while it is in fact an arbitrary regular distribution, the convergence rate will be far from optimal. It would definitely be nice to have a more robust algorithm.

Other Related Works. Prior to [Cole and Roughgarden 2014], works such as [Elkind 2007] [Dhangwatnotai et al. 2015] had the flavor of learning the optimal price/auctions from samples, but not yet fitting into the language of sample complexity.
Sample Complexity of Single-parameter Revenue Maximization

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Table II. Sample complexity bounds in [Guo et al. 2019]
quantiles and the true quantiles. This can be bounded using standard concentration inequalities. For example, suppose a value $v$ has quantile $q$. Then, Bernstein’s inequality gives that, with high probability, its quantile in the empirical distribution is approximately equal to $q$, up to an additive error of:

$$\tilde{O}\left(\sqrt{\frac{q(1-q)}{m}}\right).$$

To ensure that the error bound holds for all values, one can simply take a union bound at the cost of an extra logarithmic factor inside the square root. Intuitively, the dominated empirical distribution is obtained by subtracting this term from the quantile of each value $v$ in the empirical distribution.

Next we explain the main difference between our algorithm and those in previous works, with the exception of [Roughgarden and Schrijvers 2016]. Previous works generally pick the optimal auction w.r.t. the empirical distribution, with a distribution-family-dependent preprocessing on the sample values, in the form of truncating large but rare values and/or a discretization of the values. The preprocessing is to avoid choosing the auction based on some rare but high values in the samples. In contrast, our algorithm picks the optimal auction w.r.t. the dominated empirical distribution, without any preprocessing or any knowledge of the underlying family of distributions. The conservative estimates of quantiles by the dominated empirical distribution implicitly tune down the impact of rare but high values, simultaneously for all families of distributions.

The algorithm by [Roughgarden and Schrijvers 2016] is the most similar one to ours. They also constructed a dominated empirical distribution and picked the corresponding optimal auction. A subtle difference is that they used the Dvoretzky-Kiefer-Wolfowitz (DKW) inequality [Dvoretzky et al. 1956] to bound the estimation error of the empirical distribution and to construct the dominated empirical distribution, which on one hand avoided losing a logarithmic factor from the union bound, but on the other hand did not get the better bounds for values with quantiles close to 0 or 1 as in Eqn. 2. The latter property is crucial for our analysis. We leave as an interesting open question whether there is a strengthened version of the DKW inequality with quantile-dependent bounds. Such an inequality will improve the logarithmic factor in the upper bounds of this paper. We stress that while the algorithms are similar in spirit, our analysis is fundamentally different, as we will explain next. Importantly, our sample complexity upper bounds hold for the general non-i.i.d. case while the upper bound of [Roughgarden and Schrijvers 2016] holds only for the special case of i.i.d. bidders.

**Analysis via Revenue Monotonicity.** Our analysis consists of two components. The first one is two inequalities that lower bound the expected revenue of the dominated empirical Myerson auction on the true distribution, where the inequalities are enabled by the strong revenue monotonicity of single-parameter problems by [Devanur et al. 2016]. The strong revenue monotonicity states that the optimal auction w.r.t. a distribution that is dominated by the true distribution gets at least the optimal revenue of the dominated distribution. In particular, running the dominated empirical Myerson on the true value distribution $D$ gets at least the optimal revenue of the dominated empirical distribution $\tilde{E}$. Further, consider a
doubly shaded version of the true distribution, denoted as $\tilde{D}$, which intuitively is obtained by subtracting twice the error term in Eqn. 2 from the quantiles of the true distribution. Then, $\tilde{D}$ is dominated by $\tilde{E}$ and, thus, its optimal revenue is at most that of $\tilde{E}$. This weaker notion of revenue monotonicity is folklore in the literature and follows as a direct corollary of the stronger notion. Therefore, we conclude that the expected revenue of the dominated empirical Myerson auction is at least the optimal revenue of the doubly shaded distribution $\tilde{D}$. It remains to compare the optimal revenue of $D$ and $\tilde{D}$.

This idea is quite powerful on its own. The key observation is that $\tilde{D}$ approximately preserves the probability density/mass of $D$ almost point-wise, except for a small subset of values that have little impact on the optimal revenue. Intuitively, this is because it consistently underestimates the quantiles; in contrast, the empirical distribution has fluctuations in its estimations. Hence, $\tilde{D}$ approximately preserves the virtual values of $D$ almost point-wise, circumventing the technical hurdle faced by the second previous approach discussed in Section 2. By this idea and standard accounting arguments for the expected revenue, we can get the optimal sample complexity upper bound for regular distributions in Table II, and match the best previous upper bounds for the other three families of distributions in Table I.

**Analysis via Information Theory.** To get the optimal sample complexity upper bounds for all families of distributions under a unified framework, we need the second idea, namely, to bound the difference between the optimal revenues of $D$ and $\tilde{D}$ with an information theoretic argument. The argument consists of two claims: 1) the distributions $D$ and $\tilde{D}$ are similar in the information theoretic sense so that it takes many samples to distinguish them, and 2) we can estimate the expected revenue of any given mechanism on $D$ and $\tilde{D}$ with a small number of samples. Concretely, we will show that the Kullback-Leibler (KL) divergence between $D$ and $\tilde{D}$ is at most $\tilde{O}(\frac{n}{m})$, omitting some caveats which are explained in details in [Guo et al. 2019]. By standard information theoretic arguments, it implies that one needs at least $\Omega(\frac{m}{n})$ samples to distinguish these two distributions. For example, consider a $[0, 1]$-bounded distribution $D$ and an additive $\epsilon$ approximation. Suppose $m$ is at least $O(\epsilon^{-2})$ as in Table II. Then, we get that it takes at least $C \cdot \epsilon^{-2}$ samples to distinguish $D$ and $\tilde{D}$ for some sufficiently large constant $C > 0$. On the other hand, it takes less than $C \cdot \epsilon^{-2}$ samples to estimate the expected revenue of any mechanism on both $D$ and $\tilde{D}$ up to an additive $\epsilon$ factor. Thus, the expected revenue of any mechanism differs by at most $\epsilon$ on the two distributions; otherwise, we can distinguish them with less than $C \cdot \epsilon^{-2}$ samples by estimating the expected revenue of the mechanism. As a result, the optimal revenues of $D$ and $\tilde{D}$ differ by at most $\epsilon$.

To our knowledge, this is the first time information theory is used to show sample complexity upper bounds for revenue maximization. Previously, it was used only for lower bounds (e.g., [Huang et al. 2015]). We believe it will find further applications in studying the sample complexity of multi-parameter revenue maximization and other learning problems. We stress that our algorithm is constructive and, in fact,
can be implemented in quasi-linear time: \(^3\) both the doubly shaded distribution \(\tilde{\mathbf{D}}\) and the information theoretic arguments are used only in the analysis.

**Lower Bound Constructions.** Our lower bounds are unified under a meta construction, with some components chosen based on the family of distributions. We briefly sketch the construction below. Let the first bidder's value distribution be a point mass. She will serve as the default winner in the optimal auction. The value distribution of each of the other \(n-1\) bidders will be either \(D^h\) or \(D^\ell\). These two distributions satisfy that there is a value interval such that for any value in it, the corresponding virtual value wins over bidder 1 if and only if the distribution is \(D^h\). Both \(D^h\) and \(D^\ell\) will have an \(O(\frac{1}{n})\) chance of realizing a value in this interval. Intuitively, to find a near optimal mechanism we must be able to distinguish the bidders with distribution \(D^h\) from those with distribution \(D^\ell\). Finally, we will construct \(D^h\) and \(D^\ell\) to be similar so that it takes many samples to distinguish them. The meta construction, inspired by the hard instances by [Cole and Roughgarden 2014], can be viewed as a non-trivial generalization of the lower bound framework by [Huang et al. 2015] for the special case of single bidder.

### 4. BEYOND SINGLE-ITEM AUCTIONS

The aforementioned techniques generalize to the case when the seller has multiple copies of the item and therefore can allocate to multiple bidders. Suppose there are \(k\) copies, and bidders are unit-demand in the sense that giving a bidder multiple copies does not make her more happy than having just one copy. Further, suppose the bidders’ values have support in \([0, 1]\) and the goal is achieving an \(\epsilon\) additive approximation. Then, our techniques show that the same algorithm, which returns Myerson’s optimal auction w.r.t. the dominated empirical distributions, achieves the sample complexity \(\tilde{\Theta}(nk\epsilon^{-2})\), which is optimal up to a poly-logarithmic factor. Comparing with the bounds for the single-item setting, the only difference is a linear dependence in the number of copies available to the seller.

Finally, this sample complexity lower and upper bounds generalize to the matroid setting with rank \(k\). The above setting with \(k\) copies of the item is a special case known as the \(k\)-uniform matroid.

**REFERENCES**


\(^3\)For each bidder, it takes \(O(m \log m)\) time to sort the samples and to compute the quantiles of the empirical distribution, and \(O(m)\) time to compute the quantiles of the dominated empirical distribution, and \(O(m \log m)\) times to compute the convex hull of the corresponding revenue curve, which characterizes the optimal auction.


Justifications of Welfare Guarantees under Normalized Utilities

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It is standard in computational social choice to analyse welfare considerations under the assumptions of normalized utilities. In this note, we summarize some common reasons for this approach. We then mention another justification which is ignored but has solid normative appeal. The central concept used in the 'new' justification can also be used more widely as a social objective.

Categories and Subject Descriptors: I.2.11 [Distributed Artificial Intelligence]: Multiagent Systems; J.4 [Computer Applications]: Social and Behavioral Sciences—Economics

General Terms: Theory, Algorithms, Economics

Additional Key Words and Phrases: Fair allocation, solutions Concepts, multiagent resource allocation, mechanism design without money, computational social choice, approximation algorithms

1. INTRODUCTION

Social welfare under normalized utilities is frequently considered in the computational social choice literature. It is especially the case in multi-agent resource allocation, mechanism design without money and utilitarian voting. The welfare notions considered include utilitarian social welfare (sum of agents' utilities), egalitarian social welfare (minimum of agents' utilities) and Nash social welfare (product of agents' utilities). Most of the focus is on utilitarian welfare which goes back to ideas of Bentham [1789].

For multi-agent resource allocation under ordinal preferences, researchers have considered how well social welfare is approximated by specific mechanisms for truth-ful preferences or preferences in equilibrium (see e.g., [Filos-Ratsikas et al., 2014, Guo and Conitzer, 2010, Bertsimas et al., 2011]). The standard assumption in these papers is that utilities are normalized i.e., an agent’s sum of utilities for all items is one. The assumption has been termed as unit-sum.

Similarly, there is growing literature on implicit utilitarian voting (see e.g., [Caragiannis et al., 2016b, Lu and Boutilier, 2011, Boutilier et al., 2012]) where normalized utilities are popular. For these settings, the distortion of a voting rule is used as a measure of how good the rule is. Distortion is the worst ratio of the maximum utilitarian social welfare versus the utilitarian social welfare achieved among all problem instances. Many results concerning distortion bounds hinge on the assumption the utilities are normalized.1

In other multi-agent resource allocation problems, agents are asked to express cardinal utilities over items but these utilities are assumed to be normalized or are normalized.

1There is another stream of results on distortion that assume that utilities are induced by a metric (see e.g., [Anshelevich and Postl, 2016].)

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ACM SIGecom Exchanges, Vol. 17, No. 2, October 2019, Pages 71–75
processed to be normalized. For example, Bouveret and Lemaître [2015] state that most collective utility functions only make sense if the utilities are expressed on a common scale or normalized. Even when agent preferences are ordinal, the scoring rules used to get ‘proxy utilities’ satisfy the normalized or equal scale property (see e.g., [Baumeister et al., 2014]).

In short, both within voting and resource allocation, there is a focus on welfare under normalized utilities. When the ‘real’ utilities of agents are not normalized, it begs the question that why normalized utilities are used. The papers making this assumption either justify it as a standard assumption used in previous work or by mentioning one or two reasons. In this note, we curate and discuss justifications for welfare approaches under normalized utilities.

First Principle/Philosophical Justifications

(1) Scale invariance. Egalitarian and utilitarian social welfare suffer from being responsive to scale variance. Imposing normalized utilities is a simple way to regain scale invariance.

(2) Guaranteeing a proportion of the ultimate happiness of an individual. Another way to view scale invariance is that one is concerned less about the amount of utility achieved by an individual agent and more so about the fraction of maximum possible utility that she achieves. This concern is indeed captured when one tries to approximate egalitarian welfare under normalized utilities.

(3) Fairness. In voting, when each voter is allowed to spread a utility of 1 over the alternatives, it is akin to each voter having ‘one’ vote and ‘equal’ say. Similarly, normalized utilities are considered in resource allocation to define rules such as Adjusted Winner [Brams and Taylor, 1996]. If utilities are not normalized, and the rule is welfarist in some sense, then fairness can be undermined. For example, when considering rules that are responsive to utilitarian social welfare, the outcome is favourable to agents with the most magnified utilities. When considering egalitarian welfare, the allocation is aligned with the concerns of the agent with most scaled down utility valuations.

Justification in a common setting

(4) Probabilistic perspective. Another possible justification for using normalized utilities is using the utility as a measure of certainty of liking an item or outcome.

Technical Justifications

(5) Strategic. The reasons for fairness can also be seen as justification regarding strategic issues. Considering normalized utilities circumvents certain trivial
manipulation actions of scaling up or down of utilities. When considering utilitarian welfare normalization prevents agents from overshadowing other agents by reporting extremely high utilities. An agent who reports the highest utilities for all items would get all the items in a utilitarian social welfare maximizing solution!

(6) **Reasonable approximation guarantees for welfare.** Another reason for considering normalized utilities is also technical and somewhat ‘self-serving’. There is little hope of achieving reasonable approximation guarantees of maximum welfare or reasonable low distortion when utilities are unbounded. Therefore the normalization assumption can be seen as a trick of the trade to obtain reasonable approximation guarantees.

We have discussed reasons for considering (approximate) welfare guarantee for normalized utilities. There are at least two possible points of criticism concerning social welfare under normalized utilities. Firstly, one may negatively perceive that the welfare guarantee results only hold under the restrictive assumption of normalizes utilities. Secondly, there can be a more classical objection to a cardinal approach to social welfare itself which involves interpersonal comparison of utilities (see e.g., [Robbins, 1935]). The second issue may appear especially acute in the context of social choice settings such as fair allocation and voting which typically do not involve money. One reason for pursuing social welfare in computer science research has been that ‘without explicit optimization objective that measures the quality of outcomes, approximation cannot play a role’ [Procaccia and Tennenholtz, 2013].

We point out that by simply pursuing justification (6), one can paradoxically get a justification that avoids both criticisms. One can get a similar guarantee for a concept that does not involve interpersonal comparison of utilities and does not require scaling of utilities of any agent. This largely ignored ‘new’ justification is centered around a relaxation of Pareto optimality.

(7) **Approximate Pareto optimality under any scaling of the utilities.**

We say that utility profile\( u = (u_1, \ldots, u_n) \) Pareto dominates utility profile\( u' = (u'_1, \ldots, u'_n) \) if \( u_i \geq u'_i \) for all agents \( i \) and \( u_i > u'_i \) for some agent \( i \). Given any \( \alpha \in [0, 1] \), a utility profile \( u \) is \( \alpha \)-Pareto optimal if there exists no other achievable utility profile \( u' \) such that \( \alpha \cdot u' \) Pareto dominates \( u \).\(^5\) The concept is very natural and has been used in the context of routing games [Aumann and Dombb, 2010], probabilistic matchings [Immorlica et al., 2017] and participatory budgeting [Aziz et al., 2017].

It can be proven that any outcome that achieves \( \alpha \) fraction of the maximum social welfare under normalized utilities also satisfies \( \alpha \)-Pareto optimality under any scaling of the utilities. Suppose agents have normalized utilities. Consider any outcome that achieves \( \alpha \) fraction of the maximum social welfare. Suppose the utility profile of the agents is \( u \). Then we first claim that \( u \) is \( \alpha \)-Pareto optimal. Suppose it is not \( \alpha \)-Pareto optimal. Then there exists another achievable utility profile \( u' \) such that \( \alpha \cdot u' \) Pareto dominates \( u \). But this means that \( u \) achieves less

\(^5\)The definition is written for positive utilities but can be adapted for negative or mixed utilities.
than $\alpha$ fraction of social welfare of $u'$ which is a contradiction. We have established that the outcome achieves $\alpha$-Pareto optimality under normalized utilities. We note that $\alpha$-Pareto optimality is invariant under scaling of an agent’s utility function even when discrete outcomes are considered. The reason is that for an agent $i$, any two utility functions $u_i$ and $u_i'$ where $u_i' = \alpha u_i$ and for any social outcomes $a, b$, $u_i(a) \geq u_i(b)$ if and only if $u_i'(a) \geq u_i'(b)$. Hence, the outcome achieves $\alpha$-Pareto optimality under any scaling of the agents’ utilities.

By normalizing the utilities and achieving or establishing some bound on the maximum utilitarian or egalitarian social welfare also implies the same approximation bound on Pareto optimality for the ‘real’ utilities of the agents. Thus one can use justification (6) for technical ease but then achieve a guarantee that has more wide-spread normative appeal. The ‘new’ justification can also be used as another motivation for the projects of implicit utilitarian voting and approximate mechanism design without money. It can also be viewed as a source of corollaries for these lines of work. Taking another view, for new settings, one can directly use approximate Pareto optimality rather than particular social welfare as the social objective. A lower bound result for approximate Pareto optimality would imply a similar lower bound for utilitarian welfare.

We conclude by mentioning that the normative/axiomatic approach in traditional social choice and the quantitative welfarist approaches employed in recent computational social choice papers have been termed as distinct from each other. The use of approximate Pareto optimality provides a convenient bridge between the two approaches.

Acknowledgments
The author thanks Felix Brandt, Hu Fu, Xin Huang, Barton Lee, Ariel Procaccia and Matt Weinberg for useful comments.

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ACM SIGecom Exchanges, Vol. 17, No. 2, October 2019, Pages 71–75


Puzzle: The AI Circus
(Puzzle in honor of Tuomas Sandholm’s 50th birthday)

VINCENT CONITZER¹
Duke University

Please send solutions to the author by e-mail, with the title of this puzzle in the subject header. By agreement with the editors, the best solution will be published in the next issue of SIGecom Exchanges, provided that that solution is of sufficiently high quality. Quality is judged by the author, taking into account at least soundness, completeness, and clarity of exposition. A solution may be published even if it does not solve all cases.

The AI circus is in town. Just that all the performers are robots—no animal abuse in this circus—does not mean that there is no mischief!

The monkey has stolen the clown’s favorite red nose, and placed it on the top of the pentagonal tent (view from above: Figure 1). The clown would like to get her nose back, and can climb to the top of the tent along one of the corner sides (edges in the graph). Unfortunately for her, the monkey is still on the edge of the top of the tent; i.e., he is moving along the inner pentagon. If the monkey is at the same place (vertex) as the clown when she reaches the inner pentagon, he will throw her off. She will be fine, thanks to her inflatable clown suit, but the suit will be damaged, so she only has one attempt to get past the monkey and get her nose back. If she gets past the inner pentagon, the monkey will not follow her up and she will get her nose back. Besides climbing up, the clown can also ride her unicycle along the edge of the tent (outer pentagon). However, the decision to climb is irreversible; from that point, the only way down is to fall.

All moves are discretized: moving along an edge of either pentagon takes 1 unit of time. Note that the clown, on her unicycle, is much faster than the monkey, getting from one corner to the next in one time unit, whereas it takes the monkey two time units (in spite of being on the inner pentagon). The climb up from the outer to the inner pentagon takes \( x \in \mathbb{R} \) units of time. Both the clown and the monkey are running the latest game-solving AI to achieve their objective in this zero-sum game. There is no deadline and the clown and monkey are in no rush; the audience will just have to wait until the game plays out. Assume they start at the northernmost vertices of their respective pentagons and alternate moves, with the clown moving first. I.e., first the clown moves for one time unit while the monkey stands still; then the monkey moves for one time unit while the clown stands still; etc. One is allowed to pass, i.e., stand still for a move. Once someone starts down an edge, she has to keep going (including the clown climbing up the side). The monkey will throw the clown off if they are ever in the same place, regardless of who got there first.

¹I thank Yu Cheng, Yuan Deng, Troels Bjerre Lund, Caspar Oesterheld, and Tuomas Sandholm for helpful feedback.

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ACM SIGecom Exchanges, Vol. 17, No. 2, October 2019, Pages 76–77
Fig. 1. View of the circus tent, from the top.

(1) Suppose this is a perfect-information game: i.e., both players know where the other player is at each point in time. For what values of \( x \) can each player win?

(2) Now suppose that the monkey cannot see across the tent (but the clown has perfect information). I.e., if the monkey is at a corner vertex of the inner pentagon, there are three outer pentagon vertices (and three edges along which the clown can climb) he can see, and if he is at a middle vertex, there are only two outer pentagon vertices he can see. I.e., he can see across a face of the graph but no further. If the clown is not at one of the visible vertices, the monkey does not know where she is (other than what he can infer from what he knew about where she was before).\(^2\) For what values of \( x \) can each player win with certainty? Win with probability arbitrarily close to 1? Win with some other probability?

(3) Now suppose the monkey has perfect information, but the clown does not. (The clown can see the monkey in exactly the situations where the monkey could see the clown in the previous case.) Answer the same questions as before.

(4) Finally, suppose that neither player has perfect information. Answer the same questions as before.

\(^2\)Note that to play this variant as a board game, one would need a referee or computer (as in the game of Kriegspiel).