A General Framework for Endowment Effects in Combinatorial Markets

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"Losses loom larger than gains" — Daniel Kahneman; Amos Tversky

The *endowment effect*, coined by Nobel Laureate Richard Thaler, posits that people tend to inflate the value of items they own. Recently, Babaioff, Dobzinski and Oren [EC'18] introduced the notion of *endowed valuations* — valuations that capture the endowment effect — and studied the stability and efficiency of combinatorial markets with endowed valuations. They showed that under a specific formulation of the endowment effect, an *endowed equilibrium* — market equilibrium with respect to endowed valuations — is guaranteed to exist in markets with submodular valuations, but fails to exist under XOS valuations. We harness the endowment effect further by introducing a general framework that captures a wide range of different formulations of the endowment effect. The different formulations are (partially) ranked from weak to strong, based on a stability-preserving order. We then provide algorithms for computing endowment equilibria with high welfare for sufficiently strong endowment effects, and non-existence results for weaker ones. Among other results, we prove the existence of endowment equilibria under XOS valuations, and show that if one can pre-pack items into irrevocable bundles then an endowment equilibrium exists for arbitrary markets.

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1. INTRODUCTION

Consider the following combinatorial market problem: A seller wishes to sell a set M of m items to n consumers. Each consumer i has a non-negative value $v_i(X)$ to

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every subset of items X. The valuation functions can exhibit various combinations of substitutability and complementarity over items. Each consumer i has a *quasi-linear* utility function, meaning that the consumer's utility for a bundle $X \subseteq M$ and payment $p(X)$ is $u_i(X, p) = v_i(X) - p(X)$.

A classic market design problem is setting prices so that high social welfare outcomes arise in "equilibrium". A Walrasian Equilibrium (WE) [Walras 1874] is a pair of allocation $S = (S_1, \dots S_n)$ and item prices $p = (p_1, \dots, p_m)$, where each consumer maximizes his or her utility, and all items are allocated. As such, Walrasian prices are simple and transparent prices that clear the market. Moreover, according to the "First Welfare Theorem", every allocation that is supported by a WE maximizes the *social welfare*; i.e., the sum of consumers' valuations for their allocations, $\sum_{i=1}^{n} v_i(S_i)$ (see [Blumrosen and Nisan 2007]).

Walrasian equilibria are guaranteed to exist for the class of "gross substitutes" (GS) valuations, a strict subclass of submodular valuations (v is submodular if it satisfies decreasing marginal valuations) [Kelso Jr and Crawford 1982; Lehmann et al. 2006]. Unfortunately, the GS class is the frontier of WE existence (in some formal sense) [Gul and Stacchetti 1999]. Given the appealing properties of a WE, it is not surprising that a variety of approaches and relaxations have been considered in the literature to better understand the existence of WE in markets with more general valuations than GS.

The endowment effect. The *endowment effect*, coined by Thaler [1980], posits that consumers tend to inflate the value of the items they own. In an influential experiment conducted by Kahneman et al. [1990], a group of students was divided randomly into two groups. Students in the first group received mugs (worth of $6) for free. These students were then asked for their selling price, while students in the second group were asked for their buying price. As it turned out, the (median) selling price was significantly higher than the (median) buying price. In another experiment by [Kahneman et al. 1990], students were partitioned randomly into two groups. Individuals in one group received coffee mugs, whereas individuals in the second group received chocolate bars. The participants were then offered to exchange the goods they received with that of the other group; roughly 90 percent of both groups preferred to keep the goods they received. This phenomenon was later validated by additional experiments [Knetsch 1989; List 2011; 2003].

Until recently, the endowment effect has been studied mainly via experiments. Recently, however, Babaioff, Dobzinski and Oren [2018] introduced a mathematical formulation of the endowment effect in combinatorial settings, and showed that this effect can be harnessed to extend market stability and efficiency beyond gross substitutes valuations, specifically, to submodular valuations. Yet, they also presented a clear limitation of their formulation, namely that equilibrium existence does not extend to richer classes of valuations, such as XOS valuations and beyond. In [Ezra et al. 2020] (full version can be found in [Ezra et al. 2019]), we introduce a new framework that enables different formulations of the endowment effect, and leads to new possibilities.

Let us first introduce the formulation of Babaioff et al. [2018].

Babaioff et al.*'s formulation.* Given a valuation function v, and an endowed set X, the endowment effect induces an *endowed valuation* function, denoted by v^X, in which the value of each set Y is:

$$v^X(Y) = \alpha \cdot v(X \cap Y) + v(Y \mid X \cap Y). \tag{1}$$

This formulation captures the phenomenon that the value of the items in the endowed set increases by a factor of α, while the value of non-endowed items remains the same. Most of their results are obtained for the case of $\alpha = 2$, where $v^X(Y) = v(Y) + v(X \cap Y)$; i.e., the value of Y increases by $v(X \cap Y)$. An *endowment equilibrium* is then a Walrasian equilibrium with respect to the endowed valuations.

The main result of Babaioff *et al.* is that every market with submodular valuations admits an endowment equilibrium for $\alpha = 2$. Moreover, this equilibrium gives at least half of the optimal welfare with respect to the original valuations. On the negative side, they showed that the existence result does not extend to richer valuation classes, such as XOS valuations (v is XOS if it is a maximum over additive functions; XOS lies strictly between submodular and subadditive valuations [Lehmann et al. 2006]). In particular, for every α, there exists an instance with XOS valuations that do not admit an endowment equilibrium.

This strong negative result may lead one to conclude that XOS markets can remain unstable even under arbitrarily strong endowment effects. However, the formulation in Equation (1) is only one way to formulate the endowment effect in combinatorial settings; it is certainly not the only one. The question that drives us in our work is whether a more flexible formulation can circumvent this impossibility result. We answer this question in the affirmative and provide additional far-reaching results on the implications of the endowment effect in combinatorial markets.

2. A NEW FRAMEWORK FOR THE ENDOWMENT EFFECT

We provide a new framework that enables various formulations of the endowment effect based on fundamental behavioral economic principles. Similar to Babaioff *et al.*, we take a two-step modeling approach; i.e., a consumer has a valuation function v prior to being endowed with a set X, and an *endowed* valuation function v^X after being endowed with X. Our framework is based on the following principles:

The "separability principle. This principle (which is implicit in Babaioff *et al.*) states that the marginal contribution of non-endowed items does not change due to the endowment effect. Equivalently, the willingness to buy new items does not change, even in the case where the consumer disposes of some items from X.

The "loss aversion" principle. The loss aversion hypothesis is presented as part of prospect theory and is argued to be the source of the endowment effect [Kahneman et al. 1990; 1991; Tversky and Kahneman 1979]. This hypothesis claims that

people tend to prefer avoiding losses to acquiring equivalent gains.

That is, for any subset $Y \subseteq X$ of endowed items, the *willingness to sell* Y, is at least the *willingness to buy* Y, had Y not been endowed. Formally:

$$\underbrace{v^X(X) - v^X(X \setminus Y)}_{\text{Willingness to sell } Y} \geq \underbrace{v^{X\setminus Y}(X) - v^{X\setminus Y}(X \setminus Y)}_{\text{Willingness to buy } Y} \qquad \forall X, Y \subseteq M. \qquad (2)$$

Note that assuming the separability principle, the right hand side equals $v(X) - v(X \setminus Y)$.

In [Ezra et al. 2019] we observe that these two principles are characterized by the following structure: For every set $Y \subseteq M$, the value of Y, given an endowment of X, is given by

$$v^X(Y) = v(Y) + g^X(X \cap Y) \qquad (3)$$

for some function g^X, such that $g^X(Y) \leq g^X(X)$ for all $Y \subseteq X$.

The function g^X is referred to as the *gain function* with respect to X. It describes the added effect an endowed set X has on the consumer's valuation.

For two sets S, T, denote the *marginal value* of S gievn T by $v(S \mid T) = v(S \cup T) - v(T)$. By Equation (3), for every $Y \subseteq X$ it holds that

$$\underbrace{v^X(Y \mid (X \setminus Y))}_{\text{Willingness to sell } Y} - \underbrace{v(Y \mid (X \setminus Y))}_{\text{Willingness to buy } Y} = g^X(Y \mid (X \setminus Y))$$

As we shall see next, the term $g^X(Y \mid (X \setminus Y))$ is useful for quantifying the strength of an endowment effect.

The strength of endowment effects. A fundamental component of our framework is a partial order over endowment effects, which compares endowment effects based on their loss aversion strength, and (partially) ranks them from weak to strong. This partial order is *stability preserving*; i.e., given two endowment effects, one stronger than the other, any WE with respect to the weaker endowment effect is also a WE with respect to the stronger one.

Specifically, we impose the following partial order over endowment effects: if $g^X(Y|(X \setminus Y)) \geq \hat{g}^X(Y|(X \setminus Y))$ for all X, Y, then the endowment effect expressed by g is stronger than the endowment effect expressed by $\hat{g}$.

The Identity and Absolute Loss endowment effects. Recall that in Babaioff *et al.*'s formulation (for $\alpha = 2$), the endowed valuation with respect to X is $v^X(Y) = v(Y) + v(X \cap Y)$. In the terminology of our framework, the gain function is defined by $g^X(X \cap Y) = v(X \cap Y)$. Thus, we refer to this endowment effect as the identity endowment effect.

We now introduce a new endowment effect, that we refer to as the absolute loss endowment effect. In this effect, the gain function is

$$g^X_{AL}(Y) = v(X) - v(X \setminus Y).$$

For all subadditive consumers, this effect is *stronger* than identity . Intuitively, it can be imagined that in the absolute loss effect, a consumer amplifies the loss of a subset Y of an endowed set X by not accounting for the fact that $X \setminus Y$ remains in the consumer's hands.

Existence of Equilibria and Welfare Approximation. Recall that the identity endowment effect is not strong enough to lead to an equilibrium in XOS markets.

The following theorem shows that under the absolute loss endowment effect, every XOS market admits an endowment equilibrium.

Theorem 1. Every market with XOS valuations admits an absolute loss endowment equilibrium. Moreover, given any initial allocation S, an absolute loss endowment equilibrium with at least as much welfare as in S can be reached via a natural dynamics.

Our algorithm is inspired by and closely related to the "ascending price" auction used by [Fu et al. 2012; Christodoulou et al. 2016; Dobzinski et al. 2005], and can be described as follows: Starting from a given allocation, prices are set to be "supporting prices" of the corresponding XOS valuations (see [Dobzinski et al. 2005]). As long as there is a consumer that prefers to expand her bundle, given her endowed valuation and current prices, she receives the desired bundle, and the prices of all items (including items of other consumers) are updated to be the new supporting prices. This resolves the open problem raised by Babaioff *et al.* regarding natural ascending auctions that reach an endowment equilibrium. Note, however, that our process may take exponential time.

A direct corollary from the proof of Theorem 1 is that in every market with XOS consumers, every welfare-maximizing allocation S can be paired with item prices p, so that (S, p) is an absolute loss-endowment equilibrium. Notably, due to the stability preserving property of our partial order, this result applies to all endowment effects that are at least as strong as absolute loss. Moreover, we provide the following welfare guarantee.

Theorem 2. The social welfare of every absolute loss-endowment equilibrium is at least half of the optimal social welfare.

Theorems 1 and 2 show that sufficiently strong endowment effects enable the extension of equilibrium existence and welfare approximation from submodular to XOS valuations. Can this result be extended further? The answer to this question is more subtle. If the endowment effect can impose an arbitrarily high inflation factor that may depend on the parameters of the problem, then every market will admit an endowment equilibrium. For example, convince yourself that the endowment effect expressed by $g^X(Y) = \mid Y \mid \cdot v(X)$ leads to equilibrium existence in arbitrary markets, and moreover, can support the welfare-maximizing allocation in equilibrium. However, we show that for any endowment effect with inflation factor of $O(\sqrt{m})$, an endowment equilibrium may not exist for the class of subadditive valuations (v is subadditive if $v(X) + v(Y) \geq v(X \cup Y)$ for every X, Y; it is a strict superclass of XOS). Notably, identity and absolute loss inflate the valuation by a factor of 2.

3. THE POWER OF BUNDLING

We next study the power of bundling in settings with endowed valuations. A bundling $B = \{B_1, \ldots, B_k\}$ is a partition of the set of items M into k disjoint bundles. A *competitive bundling equilibrium* (CBE) [Dobzinski et al. 2015] is a bundling B and a Walrasian equilibrium (WE) in the market induced by B (i.e., the market where $B_1, \ldots, B_k$ are the indivisible items). It is easy to see that a CBE always exists. For example, bundle all items together and assign the entire bundle to the highest value consumer for a price of the second highest value. However, unlike WE, which gives welfare guarantees, no generic welfare guarantees apply with respect to CBE [Feldman and Lucier 2014; Feldman et al. 2016; Dobzinski et al. 2015]. We introduce the notion of endowment CBE — a CBE with respect to endowed valuations — and provide algorithms for computing endowment CBEs with high welfare guarantees for any endowment effect satisfying a mild assumption.

Endowment CBE computation. We provide a *black-box reduction* from the problem of computing approximately optimal endowment CBE to the purely algorithmic problem of welfare approximation. This result applies to every *significant* endowment effect, meaning that the gain functions satisfy $g^X(X) \geq v(X)$ for all $X \subseteq M$ (as is the case in, e.g., identity and absolute loss).

Theorem 3 [Black-box reduction for endowment CBE]

(1) *There exists a polynomial algorithm that, for every market with* submodular *valuations, every significant endowment effect and every initial allocation S, computes an endowment CBE, with at least as much welfare as in S. The algorithm runs in poly time using* value *queries.*[1]

(2) *There exists a polynomial algorithm that, for every market with* general *valuations, every significant endowment effect and every initial allocation S, computes an endowment CBE, with at least as much welfare as in S. The algorithm runs in poly time using* demand *queries.*[1]

The following corollary follows from the proof of Theorem 3:

Corollary. For every market, and every significant endowment effect, any welfare-maximizing allocation can be paired with bundle prices, that form a CBE endowment equilibrium.

4. EPILOGUE

Our work opens up a wide range of directions for future research, ranging from remaining gaps in this work to the analysis of new resource allocation problems in the presence of endowment effects.

A concrete open problem is the following: we show that an endowment effect that inflates the valuation function by a factor of $O(\sqrt{m})$ does not suffice for guaranteeing the existence of endowment equilibrium for subadditive valuations, while an $O(m)$ inflation suffices even for general valuations. Closing this gap is an interesting open problem.

More generally, many resource allocation problems can be revisited under the endowment effect, including both welfare and revenue maximization problems. Given the positive results provided in our work, we expect that our framework will find additional implications in other interesting settings. We hope that our work will inspire further discussion regarding meaningful endowment effects in combinatorial

[1]We refer the reader to [Blumrosen and Nisan 2007; Dobzinski and Schapira 2006] for formal definitions of value and demand queries.

settings, as well as experimental work that will shed more light on appropriate formulations for specific real-life settings.

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