Computing Core Payments in Combinatorial Auctions

ROBERT W. DAY
University of Connecticut
and
S. RAGHAVAN
University of Maryland

General Terms: Algorithms, Economics

Additional Key Words and Phrases: combinatorial auctions, core allocations, bidder-Pareto-optimality, constraint generation, VCG payments, proxy auctions

---

A *combinatorial auction* is one in which preferences are expressed for combinations or bundles of items, rather than just for individual items. The importance of combinatorial auctions has grown tremendously in the past 15 years, which have seen an explosion in the use of auction mechanisms in both government allocation problems and business-to-business commerce. Among the many applications of combinatorial auction research are auctions for shipping-lanes and other procurement in the private sector, as well as auctions for spectrum licenses by the FCC and the proposed auctioning of airport landing slots by the FAA in the public sector. In each of these environments, the expression of aggregate information allows the bidders to realize synergies (e.g., economies-of-scale or owning complementary items) while the auction mechanism stimulates competition, aiding the seller of items with more competitive prices.

Much of the literature on combinatorial auctions has focused on the underlying problem of finding an efficient allocation (called the winner-determination problem) which is usually $\mathcal{NP}$-hard, and the problem that the amount of information necessary to describe a bidder's preferences for all bundles grows exponentially in the number of items. Proposed solutions to these problems include the description of classes of winner-determination instances which are tractable, and the development of "bidding languages" which can express preferences more efficiently, respectively.

The problem of strategic difficulty for bidders is often ignored or down-played, however. In order to assume away the bidders' problem of determining an opti-

---

R. Day, Operations and Information Management, School of Business, University of Connecticut, Storrs, CT 06269-1041, bday@business.uconn.edu
S. Raghavan, Decision and Information Technologies, The Robert H. Smith School of Business, University of Maryland, College Park, MD 20742-1815, raghavan@umd.edu

mal bidding strategy, researchers often invoke the Vickrey-Clarke-Groves (VCG) payment mechanism which is dominant-strategy incentive-compatible, meaning roughly that each bidder might as well just bid her true value for each bundle of items. Assuming that the auctioneer will use the VCG mechanism for payment determination allows researchers to ignore the strategic difficulty a bidder would face in (for example) a pay-what-you-bid environment, allowing them to focus on tackling the $\mathcal{NP}$-hard winner-determination problem, or the development of a bidding language.

The drawbacks of VCG are numerous, however, and it is seldom if ever used in practice. These drawbacks are well-documented in the literature, but there is currently no consensus as to the best method for setting prices for a given set of bids in a combinatorial auction. Using some form of "sensible pricing," with payments less than the actual amount bid but large enough to "beat out" competitors, promises to encourage bidding with potential benefits to both the buyer (who can bid more aggressively with less risk) and the seller (who does best when the items go to those who value them the most, see [Ausubel and Cramton 1999]) . Currently, the concept of "bidder-Pareto-optimal" pricing within "the core" offers the best candidate for such "sensible prices," but these are difficult to compute. In our Sept. 2007 MS article we present a major improvement on the computation of such prices, and provide methods for a more informed selection among such price vectors when several exist.

Bernheim and Whinston [1986] characterize the solutions to the bidders' strategic problem in a pay-as-bid combinatorial auction environment; the equilibrium strategies of bidders can be described as "bidder-Pareto-optimal" points within the "core." (In Economics, a mechanism's outcome is said to be *blocked* if some collection of players can mutually benefit from rejecting the mechanism's outcome and form a preferable re-negotiation among themselves. If an outcome is not blocked it is said to be in the *core*. In the case of an auction, an allocation-payment outcome is in the core if no group of bidders can propose an alternate outcome which they each prefer and for which the seller receives more payments. The outcome is *bidder-Pareto-optimal* in the core if no Pareto improvement is possible within the core; if we lower one bidder's payment, some other bidder's payment must increase to remain in the core.)

Ausubel and Milgrom [2002] build upon this idea and take the first major step towards a practical implementation with their ascending package auction. Here, given an arbitrary collection of bids on packages, the mechanism produces a bidder-Pareto-optimal core outcome, computed as if the bids were the bidders' true values. Though the resulting auction is not guaranteed to be dominant-strategy incentive-compatible like the VCG, the strategic benefits to the bidders should be obvious; when the set of bids is not in equilibrium (i.e., some bidders are not using the best strategy) the mechanism essentially lowers the bids until the set of all bids are in some sense "what should have been bid."

To improve upon this paradigm, we provide a better algorithm for computing these payments using a constraint generation approach. An LP formulation of the core in payment space represents a direct description of possible outcomes (in contrast to the indirect approach of Ausubel and Milgrom [2002] and others), but the number of constraints of this formulation (there is a constraint for each coalition of bidders) grows exponentially in the number of bidders. Consequently, we devise an iterative constraint generation approach that adds constraints as needed. To determine which constraints should be added in the iterative approach, our algorithm solves a separation problem, which generates the most violated constraint (corresponding to the most upset coalition of bidders) for any potential solution until none exist and the algorithm terminates. The separation problem is equivalent to an instance of the winner-determination problem, leading to a pertinent theoretical result: determining a core outcome is $\mathcal{NP}$-hard whenever winner-determination is $\mathcal{NP}$-hard. Computational experiments reported in the paper indicate that this approach generates bidder-Pareto-optimal core prices very rapidly.

Further, our new direct approach allows for a more precise selection of a bidder-Pareto-optimal core outcome when several exist. We explore a few variations on how such a selection should take place and demonstrate some of the benefits of a total-payment-minimization selection criterion, including an optimal-incentives property. Computational experiments validate the applicability of the approach for auctions with as many as 128 unique items and 500 bids.

Finally, we note that since the final draft of Day and Raghavan [2007] was accepted, an implementation of this algorithm has been tested by OfCom, the British telecom authority, leading to its use in upcoming spectrum auctions in the U.K (in particular for three auctions that are expected to occur in 2008). As a variation of the techniques proposed in the MS paper, the algorithm used for the British spectrum license market employs a *quadratic* selection rule among total-payment minimizing core outcomes, guaranteeing a *unique* set of payments for any efficient solution. Researchers associated with the FCC have also tested this algorithm, and have indicated that it has shown promise for potential use in spectrum auctions in the United States.

REFERENCES

Ausubel, L. and Cramton, P. 1999. The optimality of being efficient. Working paper, University of Maryland. Jun.

Ausubel, L. and Milgrom, P. 2002. Ascending auctions with package bidding. *Frontiers of Theoretical Economics 1,* 1, 1–42.

Bernheim, B. and Whinston, M. 1986. Menu auctions,resource allocation and economic influence. *Quarterly Journal of Economics 101*, 1–31.

Day, R. and Raghavan, S. 2007. Fair payments for efficient allocations in public sector combinatorial auctions. *Management Science 53,* 9, 1389–1406.