Revenue Monotonicity in Combinatorial Auctions

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In recent work [Rastegari et al. 2007a; 2007b] we study revenue properties of combinatorial auctions. Consider a well-known drawback of the famous VCG mechanism: a seller's revenue can go *down* when bidders are added to an auction, contrary to the intuition that having more bidders should increase competition. Following an example due to Ausubel and Milgrom [2006], consider an auction with three bidders and two goods for sale. Suppose that bidder 2 wants both goods for the price of $2 billion whereas bidder 1 and bidder 3 are willing to pay $2 billion for the first and the second good respectively (see Figure 1). The VCG mechanism awards the goods to bidders 1 and 3 for the price of zero, yielding the seller zero revenue. However, in the absence of either bidder 1 or bidder 3, the revenue of the auction would be $2 billion.

We say that an auction mechanism is *revenue monotonic* if the seller's revenue is guaranteed to weakly decrease as bidders are dropped. As the above example shows, VCG is not revenue monotonic. In our work, we investigate the extent to which other strategy-proof (dominant strategy truthful) direct combinatorial auction (CA) mechanisms satisfy revenue monotonicity. Roughly speaking, our main result states that no reasonable, deterministic, strategy-proof mechanism is revenue monotonic, even when bidders are single-minded and their bundles of interest are known by the auctioneer.

What do we mean by a "reasonable" mechanism? Formally, we require mechanisms to satisfy the fairly traditional assumptions of *participation* and *consumer sovereignty* and the further assumption of *maximality with respect to at least two bidders*. *Participation* says that losing bidders should always pay zero. Note that

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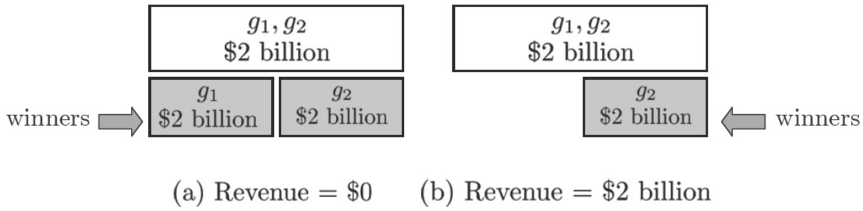

Fig. 1. An example where VCG fails revenue monotonicity: (a) all three bidders are present, (b) only bidders 2 and 3 are present

participation, as we define here, is a weaker condition than individual rationality that is usually assumed in the literature. *Consumer sovereignty* states that any bidder can win any bundle if she bids high enough. Finally, we say that a mechanism is *maximal with respect to bidder i* if the mechanism always chooses an allocation such that either (i) bidder i wins –that is i is allocated a bundle that she values above zero, or (ii) there is no item g that bidder i values more than some predefined constant amount (where the constant –"reserve price"– depends on i and g), and such that g could be (re)allocated to i without disappointing other bidders. (A bidder is not disappointed if the declared value of her allocated bundle minus g is not less than the declared value of her allocated bundle.) Intuitively, maximality ensures that the mechanism does not withhold any good or give it away to a bidder who does not value it, when there is a bidder who values it highly enough.

In the proof of our main result, we use the same bidder-bundle structure as in the example for VCG: three single-minded bidders and two goods. Bidders 1 and 3 are interested in the first and the second good respectively and bidder 2 desires both goods. Our proof works by constructing valuations for the bidders that will cause the mechanism's revenue to increase when one of the bidders is dropped. The catch, of course, is that our construction must work for *all* mechanisms that satisfy our criteria. One helpful fact (following from known necessary and sufficient conditions for strategy-proofness; see e.g., Nisan [2007]) is that strategy-proof CA mechanisms that satisfy participation offer *critical values* to known single-minded bidders. That is, given bids by the others, any known single-minded bidder i will win if she bids more than an amount that depends only on the other bidders' declarations and must pay this amount; she will lose if she bids less. Using this fact, we construct valuations by repeatedly probing the mechanism to determine the bidders' critical values given various declarations by the others. We then derive expressions for revenue when all three bidders are present and when bidder 1 is dropped. We finally show that the revenue in the two bidder case is greater than the revenue in the three bidder case—indeed, by an amount that can be made unboundedly large through the construction of the valuations.

Some interesting corollaries follow from our result. First, our proof can easily be adapted to show a similar result about revenue monotonicity when *goods*—rather than bidders—are dropped. Second, our proof can also be leveraged to show that no mechanism meeting our criteria is *false-name-proof*; this substantially strengthens a result by Yokoo et al. [2001].

Our result can be interpreted in several ways. First of all, the idea that allocations should be chosen in a way that does not "leave money on the table" is not an innocuous design decision. (Observe that this is the idea captured by the maximality condition.) Second, some issues that have been considered "problems with VCG" may in fact be properties of broad classes of CA mechanisms. Finally, attracting more bidders to an auction does not necessarily imply more competition—depending on the allocation rule, competition may actually be reduced.

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