

# VCG is the Best Anonymous Scheduling Mechanism

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## 1. INTRODUCTION

In a mechanism for scheduling on unrelated machines there are  $n$  jobs which should be assigned to  $m$  machines, where it takes machine  $i$   $t_i^j$  time units to process job  $j$  (and this is also the cost machine  $i$  incurs to process job  $j$ ). The processing times are private information of the machines. The load of machine  $i$  is the sum of processing times of the jobs  $i$  is assigned. The goal is to design a *truthful* mechanism which allocates the jobs such that the makespan, which is the maximal load over all machines, is minimized. A mechanism is truthful if it is a dominant strategy for every machine to report its true processing times.

This problem has been introduced by Nisan and Ronen [1999]. Nisan and Ronen show that the VCG mechanism achieves an approximation ratio of  $m$ , and also show that *no* mechanism, even one with unlimited computational power, can guarantee an approximation ratio better than 2. Additionally they conjectured that their upper bound is tight:

**Conjecture (Nisan-Ronen):** No mechanism for the truthful scheduling problem on unrelated machines can achieve an approximation ratio better than  $m$ .

In our paper [Ashlagi et al. 2009] we give the first strong, concrete evidence to the correctness of the Nisan-Ronen conjecture. First we briefly survey some of the most recent results about this problem:

**No real evidence so far:** Only recently Christodoulou, Koutsoupias, and Vidali [2007] were able to slightly improve the lower bound from 2 to 2.41, further improving it to 2.61 later on [Koutsoupias and Vidali 2007]. Dobzinski and Sundarajan [2008] and Christodoulou, Koutsoupias, and Vidali [2007] attempted to characterize truthful mechanisms for this problem, but succeeded in doing so only for the very limited case of 2 machines. In particular all known lower bounds are constant factors, not providing concrete evidence for the correctness of the conjecture. A different approach was taken by Lavi and Swamy [2007] who consider a special case where processing times can take only two possible values, “low” and “high”. They give several truthful mechanisms that guarantee constant-factor approximation ra-

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tios; clearly their setting is “far” from the original one.

**Our main result:** No *anonymous* mechanism for the truthful scheduling problem on unrelated machines can achieve an approximation ratio better than  $m$ .

An algorithm is anonymous, roughly speaking, if whenever two machines switch processing times, the job assignments of the two machines also switch<sup>1</sup>. Note that this is the best lower bound possible as the VCG mechanism is anonymous.

Almost all state of the art mechanisms, and algorithms for merely the scheduling problem (without incentives) are anonymous, and there is no real reason to believe that there exists a non-anonymous mechanism with a better approximation ratio. At the very least our result shows that if the Nisan-Ronen conjecture is false and there are mechanisms that provide a reasonable approximation ratio, then they must be very “strange”.

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<sup>1</sup>For the mechanism to “notice” that the machines have switched costs, we of course require that the cost vectors of the machines are unique.