Editor's Puzzle: A Dutch Dutch Auction Clock Auction

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Solutions should be sent to the editor at `conitzer@cs.duke.edu` with subject header **SIGecom Exchanges Puzzle**. The author(s) of the most elegant solution (as judged by the editor) will be allowed to publish his or her or their proof in the next issue of the Exchanges (ties broken towards earlier submissions). To make the solution accessible to a wide audience, try to minimize technical jargon in the proof. The editor will not give any feedback on submitted solutions and ignore any requests for hints, *etc.*

Sally's business has hit upon hard times, and she has decided to liquidate it. She is selling everything using a Dutch auction, one item at a time. For this purpose, she has acquired a beautiful Dutch auction clock, which starts at a price of 1 and slowly descends until somebody claims the item at the current price. In true "everything must go" fashion, the last item being sold using the Dutch auction clock is the clock itself. This particular clock was not intended for such intense use, though, and by the time the clock is itself up for sale, it is apparent to everyone that at some point during this last auction, the clock will break—unless someone claims the clock before then. If the clock breaks, it is irreparable and therefore worthless. If it is purchased before it breaks, it will keep its full value (because it will be stopped immediately when the buyer claims it, and the buyer can then take it in for maintenance, preventing its demise). If the clock breaks, this will be immediately apparent to all the bidders, so the clock will not be sold then.

There is a commonly known prior distribution over the price at which the clock will break down (if nobody claims it before then). Specifically, $W(p)$ is the probability that the clock will break down after reaching price p (at a price lower than p). For example, $W(p) = p$ indicates that this a breakdown is equally likely at every price, $W(p) = p^2$ indicates that it is more likely to break down at higher prices (earlier in the auction), and $W(p) = \sqrt{p}$ indicates that it is more likely to break down at lower prices (later in the auction).

There is some number of bidders n (e.g., $n = 2$). Also, bidders' valuations are drawn i.i.d. from a cumulative distribution $H(v)$ over $[0, 1]$ (e.g., $H(v) = v$, the uniform distribution), and this distribution is common knowledge.

Can you solve the Bayesian game that the bidders face? Try to make your solution as general as possible, but give a specific solution for the example distributions above. Does the solution make intuitive sense? Under what other circumstances is your solution useful?

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