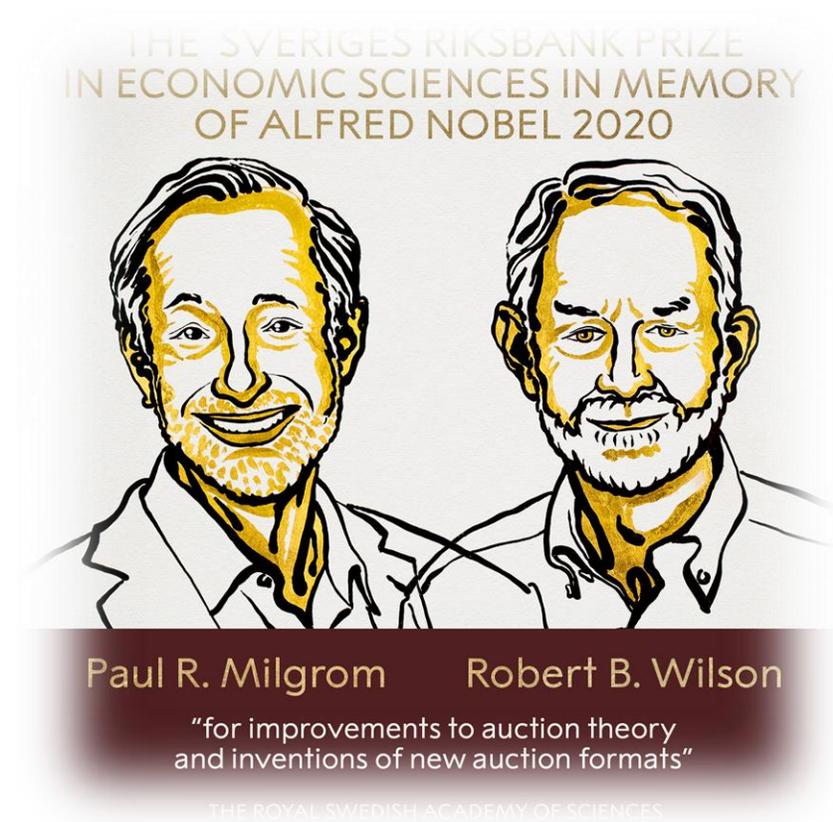# Auctions with Correlated and Interdependent Values

Part I: Contributions of Robert Wilson and Paul Milgrom

presented by **Piotr Dworczak**
(Northwestern University)

THE SVERIGES RIKSBANK PRIZE IN ECONOMIC SCIENCES IN MEMORY OF ALFRED NOBEL 2020

Paul R. Milgrom   Robert B. Wilson

"for improvements to auction theory and inventions of new auction formats"

THE ROYAL SWEDISH ACADEMY OF SCIENCES

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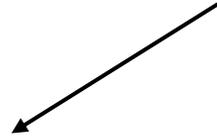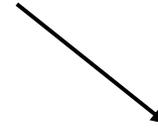**Contrast this with the canonical IPV case: independent and private values**

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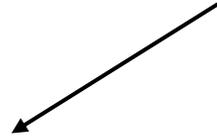**Contrast this with the canonical IPV case: independent and private values**

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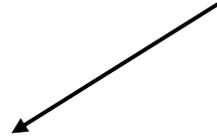
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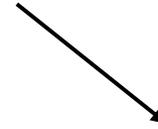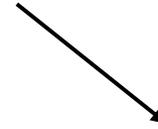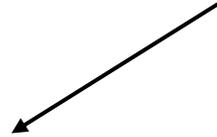**My valuation for the item being sold is not related in any way to valuations of other bidders**

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Common values: Special case when the item is worth the same amount to everyone

Outline of the Tutorial

1. **Introduction to the topic (Piotr)**

    A. **Emergence of interdependent-value auctions**

    B. **A real-time common-value auction**

    C. **The framework**

2. **Foundational papers by Robert Wilson and Paul Milgrom (Inbal)**

3. **Robustness and auction design (Brendan)**

From General Equilibrium to Common-Value Auctions

Classical idea in economics: Market prices aggregate dispersed information, thus ensuring efficient allocation of resources (Adam Smith (1776), Lèon Walras (1874) , Friedrich Hayek (1945))

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Black-box approach: These theories are non-strategic, do not feature dispersed information, and do not explain how prices are found in practice.

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But: We need a notion of the **true value of the good** which is absent from an IPV model

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Wilson's earlier work, inspired by real-life auctions, provided the necessary conceptual tools
(see also Ortega-Reichert's 1968 dissertation)

- Wilson (1967): Two major oil companies bid for an oil tract (that has the same amount of oil for both bidders). One company already owns a neighboring tract, and hence has almost perfect information. The other company only has an estimate. How should they bid?

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In the common-value auction framework, we can ask:
    Do equilibrium prices in the auction approximate the true value of the asset?

A real-time common-value auction

1. I will show you an **object with a common value**

2. You should try to assess what that value is (in US $)

3. Submit a bid in a **SECOND PRICE AUCTION** using a Google form

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Random variables $X_1, \dots, X_N, S$ have a known distribution.

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21 563 992,50 PLN ≈ 5,828,106.08 USD