

# Carbon Footprint Optimization: Game Theoretic Problems and Solutions

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We discuss four problems that we have identified under the umbrella of carbon economics problems: carbon credit allocation (CCA), carbon credit buying (CCB), carbon credit selling (CCS), and carbon credit exchange (CCE). Because of the strategic nature of the players involved in these problems, game theory and mechanism design provides a natural way of formulating and solving these problems. We then focus on a particular CCA problem, the carbon emission reduction problem, where the countries or global industries are trying to reduce their carbon footprint at minimum cost. We briefly describe solutions to the above problem.

Categories and Subject Descriptors: J.4 [**Social and Behavioral Sciences**]: Economics

General Terms: Carbon Emission, Economics, Game Theory

Additional Key Words and Phrases: Mechanism Design, Auctions, Carbon Footprint Optimization, Carbon Economics

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## 1. INTRODUCTION

World-wide, there are intense activities by all countries and global organizations to address the issues raised by climate change and global warming. A significant cause for climate change and global warming has been the green house gas (GHG) emissions and other pollutants by industries across the globe. A major contributor among GHGs is the emission of carbon dioxide and hence GHG emissions are also referred as carbon emissions. Carbon emissions are measured in terms of *carbon credits* where one carbon credit is equal to one ton of carbon dioxide ( $CO_2$ ) emitted. Standard conversion units for other green house gases are available to obtain equivalent  $CO_2$  emissions. The well known Kyoto protocol introduced the *carbon trading mechanism* to be used by global industries or organizations as a means to incentivize them for their emission reduction efforts. The basic approach for carbon trading involves the *cap and trade* mechanism. A cap and trade system is a market based approach to control pollution that allows global industries or national governments to trade surplus emission allowances after meeting the cap or limit, on those emissions. This mechanism involves two parties, (1) the governing body (or the regulatory authority) and (2) the regulated companies or organizations emitting pollution. The governing body sets a limit called the *carbon cap* on the total amount of  $CO_2$  and other green house gases (equated in terms of  $CO_2$ ) that could be emitted in a given period and will issue rights, or allowances, corresponding to that prescribed level of emissions. Companies that can more efficiently reduce carbon emissions can sell permits to companies that cannot easily afford to reduce emissions. The companies that sell the permits are rewarded while those that purchase permits pay for their negative impact.

### 1.1 Carbon Economics Problems

We have identified the following four generic problems of carbon emission management, in the context of a country or global industry or organization [Arava et al. 2010]. In the rest of the paper, we use the word agent to represent the country, industry, or organization, as the case may be.

- Carbon Credit Allocation (CCA) Problem*: Under the cap and trade mechanism, the allocation of cap to agents becomes an important problem so as to limit the carbon emissions to be less than or equal to the cap. The allocations should consider aspects of varying cost of reductions for different agents, capacity of reduction of each agent, and policy issues. We discuss this problem in Section 2.
- Carbon Credit Buying (CCB) Problem*: Agents who cannot reduce their carbon emissions to the level of cap can offset their carbon emissions by buying the required amount of carbon credits from global carbon market or so called carbon exchanges. Agents also have an option to invest in a Clean development mechanism (CDM) and Joint Implementation (JI) projects defined under Kyoto protocol. This gives rise to an interesting problem where the company has to first optimize internally and then buy the extra credits from the market keeping the procurement cost minimum.
- Carbon Credit Selling (CCS) Problem*: The agents can earn revenue by selling their surplus carbon credits, to agents that fail to meet the cap. Thus, businesses that are involved in reducing carbon emissions producing low emissions

can benefit by selling carbon credits in the market. This gives rise a problem where companies have to optimally make decisions on investments so as to be on the surplus.

—*Carbon Credit Exchange (CCE) Problem*: The CCB/CCS problem only considers situations where only buyers/sellers are interested in buying/selling carbon credits. An exchange would allow multiple buyers and sellers to trade carbon credits. This gives rise a large set of problems, similar to that of a stock exchange.

The agents involved in the above problems are typically independent companies or independent units of a company. These agents hold private information such as cost of reducing emissions, capacity of emission reduction, etc and there may not be any incentive for them to report this information truthfully. The four carbon economics problems mentioned above are therefore decision or optimization problems with incomplete information, involving strategic agents. It is required to implement a system-wide solution that satisfies desirable properties such as truthful reporting of private information, efficiency of allocation, budget balance, and voluntary participation. Clearly, a natural way of modeling and solving these problems would be through mechanism design [Narahari et al. 2009; Arava et al. 2010]. To explain this further, in the next section, we will explore the CCA problem in more detail.

## 2. CARBON CREDIT ALLOCATION PROBLEM

Consider a global industry that has multiple divisions. Each division is an independent unit of the company or a supply chain partner and has capability to measure its carbon emissions truthfully. We assume that the industry under consideration has received a cap on its total emissions from a regulatory authority (for example, the federal government). Let  $E$  be the current (or historical) total number of carbon units emitted by the industry and the cap prescribed is  $C$  units and usually we have  $C < E$ . Hence the industry has to reduce or offset  $M = E - C$  emission units. The industry wishes to achieve this by *optimally* allocating these  $M$  reduction units among its divisions. As the cost of reductions vary for different divisions, a natural objective of the allocation would be to keep the cost minimum.

The industry here plays the role of a social planner and asks each division to report its cost functions (or cost curves) for the reductions. We assume that the divisions have a finite set of solutions say  $S = \{s_1, s_2, \dots, s_m\}$ . The cost for implementing the solutions and the respective number of carbon credit reductions obtained is given by the sets  $C = \{cs_1, cs_2, \dots, cs_m\}$  and  $R = \{r_1, r_2, \dots, r_m\}$ . The solutions for carbon emission reductions can be of varying types and may use either consumable items or a new process. If a solution makes use of consumable items, it means that the currently used raw material is replaced by another raw material that is more environment friendly but is perhaps more expensive than the original material. Here we will have: if  $r_i < r_j$ , then  $cs_i < cs_j \forall i, j \in \{1, 2, \dots, m\}$ . If the set  $C$  is sorted in increasing order, then the set  $R$  will also be in increasing order. For consumable items, the cost can reduce with reductions if the regulatory authority provides an attractive subsidy on the environment friendly materials.

In the case of carbon reduction solutions using a new process, it is reasonable to assume that the solutions are to be implemented in the order given in the set and we have  $\forall i, j \in \{1, 2, \dots, m\}$  and  $s_i, s_j \in S$  and  $i < j$ , then  $r_i < r_j$  and  $cs_i < cs_j$ .

Here we will also have the set  $C_R = \{\frac{cs_1}{r_1}, \frac{cs_2}{r_2}, \dots, \frac{cs_m}{r_m}\}$  to be an increasing set, where  $C_R$  is the set for cost per unit of reduction. Also, if we apply  $s_i$  and  $s_j$  in order, then the total reduction by combined solution will be given by  $r_{ij} = K(r_i + r_j)$  where  $r_{ij}$  is the total reduction obtained and  $K \geq 1$  is a constant factor. Here the cost curve will always be an increasing curve.

In some cases, the cost curve may become constant after a certain amount of emissions are reduced. We assume that every division has a certain finite reduction capacity (limit on the amount of emission reductions that is possible).

Under the above described settings, the social planner is faced with two kinds of situations:

- Honest*: Here the individual divisions report their true cost curves. We can formulate emission reduction allocation problem as an optimization problem where the objective is to minimize the cost of reducing  $M$ .
- Strategic*: Here the divisions behave strategically and would report their true cost curves only if it is a best response for them. In this case, the social planner has to solve the problem in two steps: (1) elicit the true cost curves and (2) determine an optimal allocation to minimize the cost of reductions.

### 3. OUR CONTRIBUTIONS AND RESULTS

We have proposed algorithms/mechanisms for the emission reduction allocation problem under both the settings described above. In the *honest* case, the problem turns out to be an interesting variant of the knapsack problem and two variants of the problems have been considered: (a) with limited budget and (b) with unlimited budget [Arava et al. 2010]. In both cases (a) and (b), we have used a greedy algorithm which uses the cost curves (bids) of each division and computes the allocation vector which is shown to be optimal. The proposed algorithms can be used by companies to make their decision in budget planning, in deciding how much to invest to meet the immediate cap, how much to invest for future planning, etc.

In [Bagchi et al. 2012], we considered the *strategic* version and proposed a mechanism that a global company may use in allocating emission reductions to its different divisions and supply chain partners towards achieving a required target in its carbon footprint reduction program. The proposed mechanism is strategy-proof and allocatively efficient and uses redistribution mechanisms. We have proposed two kinds of redistribution mechanisms. The first one is based on a reverse auction where the company procures carbon reductions from divisions and then redistribution is done appropriately to reduce budget imbalance. The second one is based on an ingenious forward auction where the company sells permits to avoid emission reductions and redistribution tries to reduce the budget imbalance. We have shown that the forward auction based approach usually outperforms the reverse auction based approach although the reverse auction performs better in some settings.

### 4. CONCLUSION

Carbon credits have become highly valuable and strategic instruments of finance in the global market and it is critical for businesses to have a well thought out strategy for carbon footprint optimization to maximize the global good of the industry. Here we have described one important problem (emission reduction allocation problem).

Other immediate problems that can be formulated and solved are the carbon credit selling, carbon credit buying, and the carbon credit exchange problems. These are problems that exist at the level of an industry as well as the country or world level. We have realized that game theory and mechanism design offer an extremely promising mathematical framework for addressing various carbon economics problems.

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